Problematic
In the theory of belief function:
• Combination operation: fuse information by combining multiple evidence corpus, resulting in a summary of the different pieces of evidence.
• Distances: measure dissimilarities between pieces of evidences, which may in turn be used to evaluate source reliability to perform a fusion or combination.

Combination of evidence corpus are used in decision making, while classification/clustering the evidence corpus can be used in grouping the uncertain/imprecise objects.

How the two applications interplay? A common question:
- Can we use combination rules to compute the centroid of a group of evidence corpus?

Properties for combination/centroid calculation

Centroid of evidence corpus:

Computing the centroid is then the problem of finding the mass \( m \), defined from an arbitrary set \( \{m_1, \ldots, m_l\} \) of mass functions as:

\[
\mathbf{m} = \arg \min_{m \in \mathcal{E}(\Omega)} \sum_{i=1}^{l} d(m, m_i).
\]

Usually, to guarantee the convergence of inertia in the iteration of centroid-based clustering, the combination rule must be consistent with a metric.

- **Property 1 Metric consistency**: The combination rule \( \oplus \) and the dissimilarity measure \( d \) must be consistent. Formally, this means that the combined mass function \( \mathbf{m} \) calculated by:

\[
\mathbf{m} = \bigoplus_{i=1}^{l} m_i,
\]

should be the centroid, i.e.,

\[
\sum_{i=1}^{l} d(m, m_i) > \sum_{i=1}^{l} d(m, \mathbf{m}), \quad \forall m \in \mathcal{E}(\Omega), \; m' \neq \mathbf{m}.
\]

Least commitment principle (LCP) in TBF:

When several belief functions are compatible with a set of constraints, the least informative according to some informational ordering (if it exists) should be selected.

LCP is applied in many seminal combination rules, it required the property of **ignorance neutrality** defined as:

- **Property 2 Ignorance neutrality**: Vacuous mass functions representing ignorance are neutral elements of the combination rule \( \oplus \), i.e.,

\[
m \bigoplus_{i=1}^{l} 1_i = m, \quad \forall m \in \mathcal{E}(\Omega),
\]

where \( 1_i \) denotes a vacuous mass function.

Another property for cautiousness in an information fusion process.

- **Property 3 Idempotence**: The combination rule \( \oplus \) is idempotent if and only if:

\[
m \bigoplus_{i=1}^{l} m_i = m, \quad \forall m \in \mathcal{E}(\Omega).
\]

Indeed, Idempotence is also a necessary condition of "metric consistency".

The impossibility theorem

Given a dissimilarity measure \( \mathcal{D} \times \mathcal{D} \rightarrow \mathbb{R}^+ \) on the set of evidence corpus, there is no combination rule \( \oplus \) that satisfies the properties of **metric consistency** and **ignorance neutrality** simultaneously.

Different views on clustering evidence corpus

**A metric view**

Each evidential corpus is precisely projected into a manifold consistent with a metric. (e.g. Euclidean)

**An imprecise probabilistic view**

Each mass function is projected as interval imprecision (by interval) simultaneously.

Conclusion and significance

- Combination rules with LCP are not compatible with any metric;
- Distance-based classification/clustering over evidential corpus is dubious;
- The interpretation of the mass functions must be clarified when selecting a distance.
- Imprecise probabilistic view is appropriate with the original interpretation.

Questions as perspectives:

- Since no metric is consistent with LCP, how do we theoretically evaluate their effectiveness/correctness?
- How can we relax the properties of metric to make the impossibility possible?
- How to properly classify/cluster objects with uncertainty/imprecision?