Optimality and modularity in human movement: from optimal control to muscle synergies

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Abstract

In this chapter, we review recent work related to the optimal and modular control hypotheses for human movement. Optimal control theory is often thought to imply that the brain continuously computes global optima for each motor task it faces. Modular control theory typically assumes that the brain explicitly stores genuine synergies in specific neural circuits whose combined recruitment yields task-effective motor inputs to muscles. Put this way, these two influential motor control theories are pushed to extreme positions. A more nuanced view, framed within Marr's tri-level taxonomy of a computational theory of movement neuroscience, is discussed here. We argue that optimal control is best viewed as helping to understand "why" certain movements are preferred over others but does not say much about how the brain would practically trigger optimal strategies. We also argue that dimensionality reduction found in muscle activities may be a by-product of optimality and cannot be attributed to neurally hardwired synergies *stricto sensu*, in particular when the synergies are extracted from simple factorization algorithms applied to electromyographic data; their putative nature is indeed strongly dictated by the methodology itself. Hence, more modeling work is required to critically test the modularity hypothesis and assess its potential neural origins. We propose that an adequate mathematical formulation of hierarchical motor control could help to bridge the gap between optimality and modularity, thereby accounting for the most appealing aspects of the human motor controller that robotic controllers would like to mimic: rapidity, efficiency, and robustness.

1 Introduction

The vision neuroscientist David Marr, in his posthumous book, distinguished three levels of analysis in the field of computational neuroscience [97]. In computational motor control, the higher level ("theory") addresses questions such as: why do humans displace their limbs the way they do? what hidden goal(s) do they try to achieve? [133]. Essentially, this level of analysis seeks to explain why human movement trajectories have certain characteristics and what makes the elected movement better than another. By intuition and also by analogy with the principle of least action in classical mechanics, some researchers (including Marr himself) presumed that human behavior may be optimal in a sense that remains to be specified. Formulated in this way, the question turns out to be an *inverse optimal control* problem, a class of problems that mathematicians started to tackle a long time ago [e.g. 83] and has been recently applied to reaching [57], locomotion [27] or even flying [1]. Precisely, inverse optimal control is the process of recovering the optimality criterion (or cost function) according to which a bunch of *ex hypothesi* optimal trajectories are indeed optimal. It is an ill-posed problem and, in fact, harder than direct optimal control [8, 117]. Direct optimal control consists in *a priori* guessing the cost function and computing the corresponding optimal trajectories, which can be a tricky mathematical problem in itself in many occasions. Importantly, at this level of analysis, no assumption is made about how the brain could manage to generate these optimal trajectories or whether it constantly solves optimal control problems from scratch[1]. This type of question is left to the second level of analysis called "algorithm" which investigates how observably optimal trajectories could be generated by the sensorimotor system. To this aim, several motor control theories have been proposed such as the ones based on active inference which do not even require the specification of a cost function to explain movement generation [54]. Active inference, however, crucially requires prior knowledge about limb trajectories. As inverse optimal control precisely seeks to provide the rationale about why certain trajectories may be more valuable than others *a priori*, it can complement active inference models by informing why certain priors are

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[1] A useful analogy from classical mechanics is the principle of least action. For instance, trajectories of conservative systems are *extrema* of the *Action*, i.e. the time integral of the Lagrangian (kinetic minus potential energies), while it is hardly arguable that objects explicitly "optimize" their trajectories on purpose. In fact, finding whether a Lagrangian exists for a given system of differential equations has been the topic of numerous investigations in physics which date back to the works of Maupertuis, Euler or Lagrange. This refers to the inverse problem of calculus of variations [47] and can be seen as the analog problem of inverse optimal control. Notably, inverse calculus of variations has been used in the context of motor control to investigate the origin of the two-thirds power law [93].

used. Conceivably, through evolution and life-span development, the central nervous system (CNS) may have learned statistical descriptions of movement distributions or may have found simplified ways to trigger optimal or, say, good enough movements. By storing synergies (also called building blocks, primitives or modules[2]), it has been proposed that the CNS might have found clever ways to group and coordinate different degrees of freedom (joints or muscles) so that only a few task-related variables would need to be tuned in order to produce adequate motor patterns and efficient movements.

Modularity can be assumed at different levels: kinematic, dynamic, muscular or neural. When focusing on the muscle activation level, as in the present chapter, we talk about the muscle synergy hypothesis whose main appeal is to simplify neural motor control through timely activation of precoded groups of muscles. However, what these "building blocks" are and according to what rules they are combined remains elusive. Very often these building blocks may take the form of muscle weightings or temporal patterns, and are identified via dimensionality reduction such as PCA (principal component analysis), NMF (non-negative matrix factorization) or ICA (independent component analysis), i.e. unsupervised machine learning techniques applied to approximate motor signals (electromyographic data, aka. EMG). Currently there is still a gap between theories investigating the structure of muscle activities and theories assuming that cost functions determine limb trajectories. Yet, the crucial pivot to both approaches is musculoskeletal modeling. In inverse and direct optimal control, a model of the musculoskeletal system is required because optimally driving a system requires some dynamical description of it (e.g. rigid body dynamics with more or less advanced models of musculotendon complexes). Musculoskeletal models are also required to test the modular control hypothesis in muscle space, especially for assessing the effectiveness of the extracted synergies (from EMG) in controlling the musculoskeletal system; i.e. feeding back the EMG-based synergies into the controlled system, which is rarely done in motor control studies.

A growing number of studies are interested in investigating the links between optimality - at the highest level of analysis - (according to Marr's taxonomy) and modularity - at the second level of analysis - in order to evaluate whether optimal (or at least good enough) trajectories can be obtained from the combination of a limited number of genuine motor modules. Whether or not the algorithm

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[2] In this chapter, the terms synergy, primitive, module or building block are loosely treated as synonyms and will be used interchangeably. In the literature, a precise mathematical definition specifying the exact nature of each term is generally lacking. Different authors may thus have their own conception regarding the meaning of each term.

used by the CNS to trigger "good enough trajectories" truly relies on this kind of synergies is an open question but evidence is usually sought by tackling Marr's third level of analysis, that is, the "implementation" level. In particular, finding synergies of a neural origin would strongly support the theory. However, the nature and shape of the building blocks to be found within the CNS critically depends on the hypotheses made by the framework used to infer them. Therefore, research at the implementation level is linked to the algorithmic level; i.e. studies of the neurophysiological underpinnings of modular structures are conducted according to a predefined model of modularity.

In this chapter, we will review and discuss optimal and modular control theories and underline their strengths and limitations. It is worth noting that the three levels of analysis put forward by Marr are complementary but address conceptually different questions. Therefore, the interaction and links between the first two levels will be discussed. In Section 2, we will review the inverse optimal control approach which aims at deciphering the underlying high-level principles of motor control. In Section 3, we will review works related to the muscle synergy hypothesis that aims to assess the structure of muscle activity patterns from a dimensionality reduction perspective. Section 4 is dedicated to discussing the links between the two approaches and, finally, perspectives for future research are given in Section 5.

2 Optimal control hypothesis

2.1 Direct and inverse optimal control approaches to motor control

The inverse optimal control problem was first considered in a seminal paper by Kalman for linear-quadratic problems [83] and extended in [108]. In motor control, few studies employed such an inverse approach to tackle the motor planning problem until recently. This is not to say that optimal control has not been used; on the contrary it has been applied extensively mostly since the 80s and has emerged as a leading theory in the human movement control literature [49, 144]. The work of [106] is for instance characteristic of the classical (direct) approach that was initially employed: several costs were tested and compared to find the best descriptor/predictor of human motion data. Then, the researcher suggested that the elected cost constituted an explanatory principle of biological motion. Since that time the computational theory of motor control has developed and become very popular. An account of all the main ideas and concepts like "optimal feedback control" and “internal models” can be found in specialized reviews [144, 129, 45, 130, 127]. The success of this computational theory of motor control can be explained by two main reasons: first, once a cost function is defined, it captures everything about the possible trajectories of the system, thereby implementing an elegant dimensionality reduction from the infinite number of potential trajectories to cost functions [10]; second, it is readily implementable in artificial systems because the very same language is used by control theoretists and roboticists. The point we want to make here is that most studies tended to *a priori* choose a cost function and to test its predictions subsequently. Generally, they tried to match the standard LQR (linear-quadratic regulator) or LQG (linear-quadratic Gaussian) designs to make the problem easier to solve [84, 145]. Interestingly, these simple models were sufficient to explain several motor control phenomena (two-thirds power law, Fitts’s law, [66]). However, the true nature of the question “why do we move like that?” better conforms to a reversed process, which brings *inverse* optimal control into play. An illustration of the difference between inverse and direct optimal control is given below (Eq. 1):

$$\begin{array}{ccc} \text{observed trajectories} & \xrightarrow[\text{control approach}]{\text{inverse optimal}} & \text{cost function} \\ \text{cost function} & \xrightarrow[\text{control approach}]{\text{direct optimal}} & \begin{array}{c}\text{compare observed vs.}\\ \text{predicted trajectories}\end{array} \end{array} \tag{1}$$

The need for inverse optimal control arises from the need to identify the most plausible (among all possible) cost functions. Mathematically, a cost function may take the form $J(u) = \int_0^T h(x, u, t)dt$ where x is the controlled variable (e.g. system state such as velocity, position etc.), u the control variable and t is time[3]. If a cost function h accounts for some data, nothing precludes another cost of a different nature to perform equally well or even better. This has been at the origin of some controversy because in many occasions divergent costs were found to account for planar point-to-point reaching movements. Innovative paradigms are thus required to disambiguate these candidate costs. So, the following question arises: which of these candidate costs is really relevant to human motor control? This situation is exemplified in Figure 1.

Inverse optimal control is a difficult problem because it is primarily ill-posed unless very careful conditions are imposed. Therefore, to successfully study inverse optimal control problems, not only

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[3] In this chapter, we consider only integral costs for simplicity but we could easily add a terminal cost in all the optimal control problems.

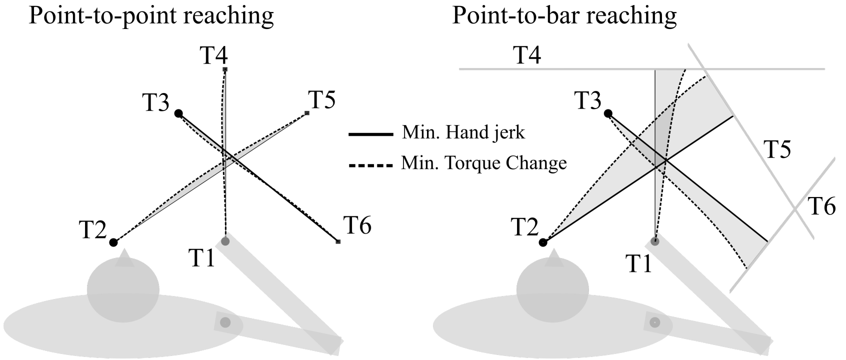

Figure 1: Predicted hand trajectories for two influential motor costs during planar reaching movements. *Left panel*: when reaching to a target point, the minimum jerk [53] and torque change [159] models predict highly similar and therefore equally plausible trajectories, whereas the two models are very different in nature (kinematic cost for the former and dynamic for the latter). *Right panel*: when replacing the point with a line or a bar, the two costs lead to different trajectories, and may thus be distinguished empirically. Shaded areas indicate the difference between the trajectories predicted by the two models. Taken from [15].

mathematical sophistication but also smart experimental designs are needed to distinguish putative cost functions.

2.2 Example of inverse optimal control results

In 2008, a necessary and sufficient condition of optimality for fast enough arm reaching movements was identified [13, 12, 57]. Namely, periods of simultaneous inactivation of opposing muscles - during the movement - were shown to be equivalent to the minimization of a cost including a term like the integral of the absolute power of muscle torques (termed "absolute work"). The strength of this result was the mathematical proof of an "equivalence" between a singular motion feature (muscular inactivation) and a cost feature (non-smoothness of the cost). Initially, the authors used a direct approach by guessing what could be a physically-relevant cost function [13]. Later, the mathematical analysis allowed to establish a powerful inverse optimal control result, whereby the authors could infer certain properties of the cost function right from experimental observations [57]. In practice, the work was guided by a different intriguing experimental result: the kinematics of vertical arm movements was shown to depend on its orientation relative to gravity vector, therefore suggesting a potential imprint of gravity on human movement [113, 63, 58]. Since then, a series of papers has shown that upward movements differed from downward movements of equal duration and amplitude (including one-degree-of-freedom motions during which only the sign of gravity effects change with movement direction; i.e. assists/resists the acceleration of downward/upward movements respectively) in that the acceleration duration was shorter in the upward compared to the downward direction [114, 63, 92, 32, 58, 128, 60, 165, 151, 61, 73]. These directional asymmetries have been observed by independent groups of researchers so consistently, including in microgravity experiments where they were actually shown to progressively vanish, that we could now employ the term “law of asymmetries” to refer to this phenomenon. In particular, minimizing the absolute work of muscle torques has been shown to robustly reproduce these vertical asymmetries for fast enough movements, despite differences in initial postures [12], upper limb’s segment [60] or gravito-inertial context [61]. This led the authors to suggest an optimal integration of gravity force during motion planning where gravity torque can be utilized to drive the limb during rapid movements. This interpretation is corroborated by EMG analyses showing that, during a rapid movement, the (phasic) activity of anti-gravity muscles is clearly lower than the (tonic) activity that would be needed to maintain a static arm posture in the same position, in certain movement phases[4] (e.g. [19, 52, 38, 39, 124]).

Previously, the absolute work was shown to be one of the potential ingredients of a more general cost function underlying human movement [12]. Various researchers had already proposed other cost functions as well as the idea of composite costs which was explicitly tested in [15]. The idea of composite costs proposes that multiple complementary criteria shape human trajectories. For instance, motion smoothness and energy expenditure are complementary criteria as minimizing one may be detrimental to the other. This mixed nature of cost functions has been observed in various tasks such as reaching [15, 162, 61], landing after a jump [167] and walking [166]. At this point, it is important to classify cost functions in two categories: the first category, “subjective” costs, depends

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[4] It is generally assumed that the muscle torque τ acting at a given joint can be split into two terms such that $\tau = \tau_{stat} + \tau_{dyn}$, where τ_{stat} is a static term which only depends on the system position and τ_{dyn} is a dynamic term which depends on its velocity and acceleration [72, 7]. Gravitational torque is part of the static term which may also include other terms like elastic forces. On this basis, researchers have proposed to split EMG activity into tonic and phasic components (e.g. [52]). To clarify our purpose, let us consider a single-joint upward movement here. If the static torques were to be compensated at all times, a phasic activity of the agonist muscle should come on top of its tonic activity during the entire motion duration. On the contrary, if the agonist EMG signal is found to be below its corresponding tonic level, it may suggest that gravity is not just counteracted but utilized as a driving force. This lack of tonic activity, already observed - but not fully considered - in several studies, actually echoes the inactivation principle mentioned in the main text. If observing proper inactivation may be tricky due to multiple factors such as the noisiness of EMGs, the predicted briefness of the phenomenon and the requirement of being under well-suited conditions of speed and amplitude, this lack of compensation of gravity torques, clearly apparent in EMG data, is additional evidence for an energy-related use of gravity in fast reaching movements.

on the subject's choice; the second category, "objective" costs, depends on the task specification[5]. In general both objective and subjective costs are relevant to motor control. Forcing a subject to reach very fast to the target, tracking an imposed trajectory or freezing a joint are examples of objective costs. Yet, even with such constraints, most motor tasks are still redundant because there is an infinity of ways, i.e. motor control solutions, to perform them. Subjective costs come into play to resolve all residual redundancy and provide the rationale about why the task is eventually performed as it is. Here, we want to underline this last statement. Reproducing or trying to explain empirical data with an optimal control model does not compulsorily mean that the nervous system solves an optimal control problem, especially not constantly and from scratch. It only means that the motor solutions the nervous system has developed, whatever the time scale, are advantageous in a sense that has been delineated.

According to the objective/subjective dichotomy, one may suggest that, in order to eliminate the effects of objective costs in the identification process, experimenters should give maximal freedom to the participant and put as little task constraints as possible. In other words, considering highly redundant tasks with very few instructions given by the experimenter may help understanding the fundamental principles humans prefer to rely on. In goal-directed tasks, there are two ways to make a task more redundant: by adding intrinsic or extrinsic degrees of freedom. First, let us consider a pointing task where the only objective constraint is to point with the fingertip toward the target. The target location defines 3 constraints (in the 3-dimensional space) but to make the task very redundant we could consider adding other degrees of freedom. For instance, asking a participant to perform a whole-body reaching task would be advantageous to emphasize subjective motor decisions, however, it would also make the optimal control problem and musculoskeletal modeling harder to solve computationally. An other alternative is to reduce the constraints imposed by the target itself. Instead of a dot, asking a subject to point to a line, or a surface, makes the task redundant even with a simple two-link arm model (e.g. Fig. 1). The appeal of this second approach is to make explicit the *choice* problem faced by a subject when planning a movement[6] (in this type of task, the subject has to select an end point on his/her own). How a subject moves when asked to point toward a line (not a point) is instructive and has been studied in a couple of papers [16, 105, 17, 150]. This type

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[5] This terminology is borrowed from [88]. In [127], the terms internal and regularization are used for subjective costs while the term task-based is used for objective costs.

[6] Remarkably, motor control has been conceived as a true (motoric) decision-making problem recently [164].

of protocol has been used to investigate eye-hand coordination [17], posture-movement coordination [69] and the use of interaction torques and velocity dependence of cost functions [162, 161].

Hitherto, all the above-mentioned modeling studies have considered that movement time was known[7]. However, motor redundancy is not only spatial but also temporal. Why are some subjects faster than others or what determines someone's movement vigor[8] are questions that were addressed more recently [132, 134, 76, 11]. It turns out to be an inverse optimal control issue as well, if one considers the existence of a cost of time [11]. Assuming a linear separability of the temporal and spatial cost functions, i.e. $J(u) = \int_0^T h(x, u, t)dt = \int_0^T g(t) + l(x, u)dt$, it was shown that it is possible to accurately recover $g(t)$ for different times t in a free-time optimal control setting [11]. In particular, this approach requires having knowledge of both the subjective and objective terms of the trajectory cost $l(x, u)$. Given these assumptions and different simple models of the musculoskeletal plant, it was also found that the cost of time exhibited a sigmoidal growth. This, therefore, raised questions such as: how would the CNS proceed to depart from its spontaneous speed choice when speed instructions are given to a subject (such as "move fast")? A complex interplay between trajectory and time costs, but also between objective and subjective costs, may be hypothesized. Indeed, it was shown that speed instructions were most plausibly captured by adding objective trajectory criteria to $l(x, u)$ [82] rather than modifying $g(t)$, and that subjective trajectory costs appeared to be quite insensitive to speed instructions [162]. Overall, time, energy/effort, smoothness but also accuracy may be relevant to biological motion planning and control. The relative weights associated to each cost element may depend on the task characteristics. The multivariate aspect of the cost function and the addition of task-dependent costs, however, make the identification problem quite hard, even though these results may truly capture some fundamental high-level goals of the motor system.

2.3 Remarks regarding optimal control theory

In general, posing an inverse optimal control problem remains difficult as it requires several choices. Indeed, both devising the class of cost functions and the dynamical system to use is a modeling

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[7] When we say that movement time is known, modeling-wise, we mean that time is set by the user (often it is taken from experimental data). Therefore, time is an input to the model. Note, however, that time can also be a free variable that emerges from optimization just as the limb's trajectory does [138].

[8] Vigor loosely refers to the speed, extent or frequency of movement [48]. It is often characterized by relationships between amplitude and velocity or duration.

choice. First, the choice of which functional space of cost functions to consider is mostly left to the experimenter. Once such an infinite dimensional space is chosen, however, a sort of discretization is required to make the problem numerically tractable and to only have a finite number of parameters to be inferred in practice[9]. One may distinguish basis costs that are chosen for mathematical reasons (e.g. polynomials) from those that are chosen for their physical/biological meaning (e.g. energy, effort...). While the former costs can be good for fitting or reproducing anthropomorphic motion, they do not allow to explain it and, thus, do not answer Marr's top level question which constitutes the main appeal of the use of optimal control for motor neuroscience. Second, the choice of the (sensorimotor) level of investigation is a related concern. It specifies the dynamical system under consideration, thereby constraining the variables that can be included in the cost function. Overall, the CNS has a hierarchical organization from task space to muscle space. Thus, the CNS may first care about what happens in task space and work with a simplified model of the musculoskeletal dynamics. Accordingly, cost functions could be defined at a kinematic level, at a dynamic level or could even attempt to minimize the overall motoneuronal activity. What level of description/investigation is best suited remains uncertain. Nevertheless, it seems that, in many cases, we can get reasonably good predictions of human movement trajectories by simple models that capture the essence of the system being controlled. For example, rigid body dynamics with some very basic muscle dynamics implementation are sufficient to capture several important motor phenomena as discussed above. Yet more involved models of the musculoskeletal apparatus have been considered too [94]. While this is a valuable and complementary approach, this raised questions regarding the confidence one can put in the "optimality" of the solutions and in the dependence of the solutions on the relative uncertainty about the model parameters such as muscle time constants, pennation angles, length/velocity force dependencies, the physiological cross-sectional area and so on, and which are difficult to know precisely for a given individual. Solving an optimal control problem can actually be a tricky task (doing the optimal synthesis[10] even for simple problems may illustrate this [117]). Quite often, we may only find sub-optimal solutions and get stuck in local minima when using numerical tools. Therefore, working with very high-dimensional and non-linear

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[9] For example, a researcher might decide to work in the space of cost functions that depend on position and speed variables, or might wish to include acceleration variables (e.g. [27]). Other assumptions could be made such as working with polynomials (e.g. [139, 115]). However, a numerical implementation would necessitate restricting to some degree n or working with a finite number of basis costs belonging to the function space under consideration.

[10] Formally, this is the set of all the optimal trajectories joining any initial state to any terminal one.

systems may be challenging also for this reason.

We must mention that, like with any theory, the question of falsifiability must be asked. Because inverse optimal control is a form of data fitting (at least its numerical implementation), we shall always find some functions that fit a given set of data. Thus, special care must be taken regarding overfitting: Occam's razor principle should be applied for numerical inverse optimal control especially when composite costs (i.e. combinations of multiple cost functions) are considered. If the CNS truly relies on certain composite costs and if one imposes the constraint that a limited number of costs should explain a variety of tasks, falsifiability could then be addressed. This would suppose that the same cost functions should be relevant to a variety of tasks in the sense of generalization and cross-validation. If, in contrast, a given mixture of costs allows to account for motor performance in a given task but not in other tasks, then it would mean that the model must be modified or even reconsidered more globally.

At last, some authors have argued that motor control is "good enough" instead of really optimal [96]. Actually, saying that a system is not behaving optimally can be considered a stronger claim than saying it is behaving optimally because it is always possible to find a cost replicating a given experimental trajectory (a trivial - admittedly meaningless - counterexample would be a cost tracking the specific trajectory to be reproduced). Yet, we agree that being good enough may be sufficient for the sensorimotor system, especially if one thinks of the existence of muscle synergies or primitives that restrain the repertoire of possible motor commands. In any case, saying that a behavior is good or favorable implies that it offers some advantages against others, which can be theoretically translated in terms of cost functions. Even though some behavioral strategy is a local, not global, optimum [55, 123], it is nevertheless an *extremum* of a certain cost function (e.g. satisfying the necessary conditions given by Pontryagin Maximum Principle, [117]), which would be useful to characterize. In the end, saying that something is good enough or even optimal (reciprocally not good enough or not optimal) makes no sense without adding "with respect to" some well-defined cost function[11].

Other theories such as the passive motion paradigm [101] have also been opposed to optimal control. The core idea in this paradigm is to replace cost functions with endpoint force fields and

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[11] A useful biomechanical analogy would be to talk about the "moment of a force" without precising the fixed reference point with respect to which it is calculated.

assume that the mechanical system is moved by a virtual force acting at the level of the end-effector (e.g. like strings moving a puppet's limbs). This is seen as a means to maximally exploit the spontaneous dynamics of the musculoskeletal system. However the nature of the underlying force field has to be given (choosing some free parameters) and, once one is chosen, it could still be interpreted as arising from a certain optimality criterion (e.g. optimal feedback control arising from LQR/LQG settings typically leads to static or time-varying force fields, in Cartesian or joint space depending on the control system under investigation). The same point could be made regarding dynamical system theory [158, 87] as, once a control law is defined, the musculoskeletal system may appear to be self-organized and governed by some ordinary differential equation. Inverse optimal control theory mainly seeks to justify why certain force fields or dynamical attractors would be utilized rather than others. Nonetheless, relying on learned dynamical patterns, or basis force fields, can be viewed as an efficient algorithmic way to solve the degrees of freedom problem and to generate effectively coordinated limb movements [103]. In our view, this level of investigation rather addresses Marr's second level of analysis (algorithm).

From a robotic perspective and towards an anthropomorphic motion factory, inverse optimal control is also appealing as it may allow roboticists to produce an infinity of human-like movements from a given cost function. Indeed, cost functions (together with a model of the plant) are able to plan movements that have not been tested or encountered previously. Yet, the difficulty to resolve quite involved optimal control problems in real time for robots with numerous degrees of freedom limits the appeal of its generalization power. Besides classical linear-quadratic formulations, we lack ways to quickly solve such problems. Model predictive control might partly resolve this issue. Although this is an increasingly popular approach, it remains however unknown whether its control architecture is biologically plausible [99]. Modularity may help efficiently solving optimal control problems in an ecological way, however further research efforts are needed to figure out how the CNS actually implements the control of limb movement.

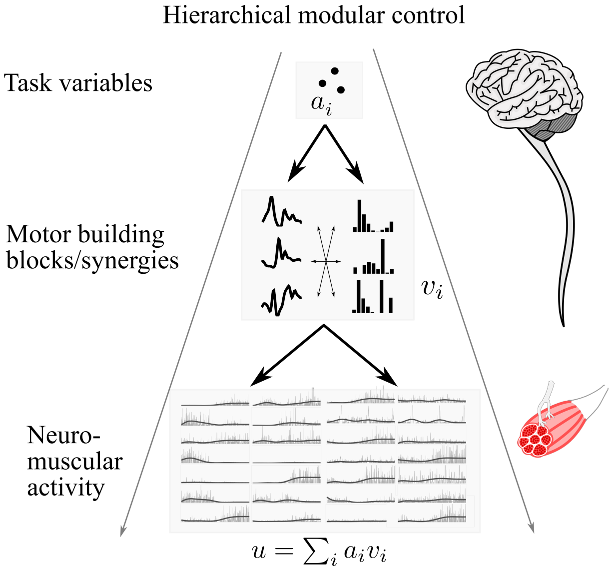

Figure 2: Illustration of the hierarchical modular control architecture at the muscle activation level. The scheme goes from task parameters (in a lower-dimensional space yielding some activations a_i), via combinations of precoded and stored motor modules (here loosely denoted by v_i), to neuromuscular inputs (in a higher-dimensional space, e.g. $u = \sum_i a_i v_i$ for a linear model). Dependencies on time/state and distinction between vector/scalar quantities are not specified on purpose. The nature of the modules and their combinations depend on the underlying model: they can be temporal waveforms, vectors of muscle activation ratios, spatiotemporal activity profiles, feedforward and/or feedback elementary control actions, and can be combined in a linear or nonlinear fashion etc. In all cases, their task-dependent modulation is assumed to account for the formation of genuine muscle patterns.

3 Modular control hypothesis

3.1 Hierarchical modular control approach to motor control

Another significant body of the motor control literature has focused on the idea of compositional or modular motor control, such as the muscle synergy hypothesis on which we focus in this chapter. This body of literature suggests that the CNS stores certain muscle synergies and is able to combine them adequately to generate a motor command that would allow accomplishing a given motor task. A useful metaphor to illustrate the concept of modularity for spatiotemporal motor signals is to depict movement generation as music playing. Music is created by combining "modules" such as a melody (notes) and a rhythm (tempo). Similarly, coordinated movement may be the outcome of the combination of such stereotyped modular structures. The idea of hierarchical modular control is illustrated in Figure 2.

The computational appeal of this theory is that, if pre-coded invariant modules can be used as motor building blocks, the CNS would only have to coordinate them to execute consistent movements, thereby simplifying the control problem by, *de facto*, implementing a dimensionality reduction. Therefore, the aim is to understand both the representation of motor commands and according to which algorithm the CNS may set them up (the second Marr's level of analysis). More generally, at this level of analysis, one would want to elucidate how the CNS builds motor commands that generate good (enough) limb trajectories (which is not necessarily via muscle synergies but could relate to active inference or other means as mentioned above).

A first step in deciphering whether and how motor building blocks are stored within the CNS consists in defining a plausible modularity model. This is again a modeling choice. The model parameters can then be inferred from experimental data and correlated with their putative neural underpinnings. To implement this first step, several models of motor modularity have been proposed in the literature, mainly differing in their assumptions about a) what quantities are stored by the motor system as invariant "modules" which can be reused in different movements and b) what quantities are determined by the descending neural motor commands to recruit the modules in single trials [156, 79, 36]. Modules have been hypothesized to represent spatial, temporal or spatiotemporal invariant patterns in motor signals [43, 5, 26]. It is worth mentioning that modularity has been assumed to exist at different levels of the motor hierarchy (e.g. kinematic, [126, 14]; dynamic, 140, 25, 124; neural, [112, 28]) with most of the studies placing modularity at the muscle activation level [155, 142, 143, 34]. Regarding the mathematical formulation, models of motor modularity are typically linear because it was found that the force fields resulting from the co-stimulation of two spinal loci were linear combinations of the individual force fields [104, 20]. Thus, the algorithms used to identify putative modules are commonly based on linear dimensionality reduction methods such as PCA or NMF and the extracted modules are assumed to be combined by feedforward motor mechanisms. Recently, techniques for the identification of modules of feedback nature have also been proposed for isometric tasks [118]. Hence, modeling choices include in general both the nature of the modules (spatial, temporal, feedforward, feedback etc.) and the associated combination rules (linear or not).

At the second step of this approach, motor signals recorded during a large number of motor tasks have been successfully fitted by modularity models. This was interpreted as evidence that performance of such motor behaviors relies on motor modules (e.g. reaching [38, 157, 26, 43, 102]; grasping [110, 18, 163]; walking [78, 79, 80, 46, 91, 21]; pedaling [75, 74]; reflex movements [156, 22, 116, 37]; postural tasks [154, 152, 153, 29, 141] etc.). Furthermore, modular structure in muscle activity has been shown to be preserved or adjusted after different types of brain injuries [24, 23, 65, 120, 121, 30].

At last, in order to probe the neural basis of modularity, the model parameters learned from the motor signals can be used as proxies of modular mechanisms, so their correlations with neural signals can be investigated. Several studies have identified potential neural bases of modules or their activations, both at the cortical level [71, 112, 111, 86] and in spinal structures [67, 68, 64, 125]. Crucially, this approach can serve to disentangle a) the nature and structure of modules, thereby informing the design of suitable modularity models and b) the level of the neural hierarchy where modules may be encoded, thereby addressing Marr's third level of analysis (the neural implementation). In this vein, Kargo and Giszter [85] showed that, at the spinal level of motor organization, premotor pulses (i.e. temporal modules) are more likely to be encoded than time-varying synergies (i.e. spatiotemporal modules). Also, Roh and collaborators [119] showed that medulla and spinal cord are sufficient for the expression of most (but not all) muscle synergies (i.e. spatial modules), which are likely activated by descending commands from supraspinal areas.

3.2 A task-space perspective to modularity

While many studies have tested modularity models based on whether they reconstruct the recorded muscle activation patterns for a number of task conditions using a limited number of invariant modules (input space assessment), recently a regain of interest to relate modularity to task space has been observed. A recent review of literature emphasizes this shift of paradigm [5]. As stated above, muscle synergies are typically extracted from recorded EMG data using unsupervised algorithms. The variance accounted for (VAF) or R^2 (R is the correlation coefficient) values are computed to evaluate the overall data approximation performed by the dimensionality reduction. However, this assessment has some limitations. First, VAF and R^2 values are somewhat arbitrary and defining an absolute threshold that indicates what is good fitting is a sensitive subject. Second, the musculoskeletal system being largely nonlinear, small errors in input space can lead to large errors in task space and undermine task achievement. Therefore, we should evaluate how putative motor synergies and their activations relate to task parameters. This idea has recently been put forward by many authors [6, 40, 42, 41, 135]. Mostly three approaches were taken: (1) using isometric tasks for which a virtual mapping from muscle space to task space can be defined by the experimenter, (2) quantifying the extent to which task identity is encoded in synergy-space so as to assess whether the way synergies are activated unequivocally determines the task at hand, as postulated by the theory, and (3) grounding on realistic musculoskeletal models to test whether experimentally-driven synergies can effectively be used for control. In a series of papers, de Rugy and collaborators evaluated "the usefulness of muscle synergies [...] in terms of errors produced not only in muscle space, but also in task space". They showed that even for what could appear as a relatively high VAF, control with muscle synergies could lead to unacceptable errors in task space. Delis and collaborators argued that single-trial task decoding/information techniques should be used to evaluate whether modularity can guarantee task performance in single trials [44]. The rationale was the following: if the performed movement cannot be discriminated in the reduced-dimension synergy space, this would cast serious doubts about the effectiveness of the proposed hierarchical modular control scheme (indeed, it is possible to get a large VAF with a low decoding score, which would invalidate a modular decomposition although it is doing a good job at reducing dimensionality). The authors concluded that complex and comprehensive data sets should be considered in general to conduct such analyses, in the spirit of large-scale neuroscience endeavors [56]. This is a relevant approach as it was shown that the number and efficiency of muscle synergies depends on the scope of the original database and on the complexity of the tasks under investigation [40, 136]. Another way to test the modular control hypothesis is to build an accurate musculoskeletal model, as done in [107, 6]. The authors of these studies applied this technique for walking and essentially showed that EMG-based synergies are rough starting point solutions that need to be fine-tuned to elicit adequate walking patterns.

Hence, the analysis of motor modularity should ideally consist of a closed loop between the recorded motor signals and their associated limb trajectories. This can be summarized as follows (Eq. 2):

$$\begin{array}{ccc} \text{real muscle patterns} & \xrightarrow[\text{engineering}]{\text{reverse}} & \text{extracted muscle synergies} \\ \text{extracted muscle synergies} & \xrightarrow[\text{musculoskeletal dynamics}]{\text{forward}} & \text{reconstructed trajectories} \end{array} \tag{2}$$

The first step allows to extract the potentially stored modules via some machine learning technique. To this aim, it is likely that switching from unsupervised to supervised learning algorithms taking into account the underlying biomechanics and trajectories in task space could lead to more advanced synergy models. However, the choice of unsupervised versus supervised learning algorithms is often neglected in practice because the simplicity of standard methods such as NMF is preferred. The second step consists in "playing back" the synergies into the musculoskeletal apparatus to test the produced motor behavior and evaluate the proposed control model in task space by analyzing the reconstructed limb trajectories.

3.3 Remarks regarding modular control theory

In reality, the questions of what the bricks of motor commands are, what model should be used to describe them and how motor commands are combined have not been resolved yet. It is likely that the models currently used are too simple to define a suitable framework from a control theory standpoint and to allow finding explicit neural correlates of the putative modules. In particular, most EMG-based identification models do not consider feedback processes whereas muscle synergies or coordination patterns may be posture-dependent as suggested in [118, 123] and, therefore, the state of the biomechanical system should possibly be taken into account during synergy identification. Another restriction is that current synergy extraction algorithms usually require equal movement durations across trials to simultaneously extract synergies from different movement conditions, which is not the case of real data. At last, except in some studies where multiple layers of modularity have been considered, thereby leading to nonlinear reductions of dimensionality [10], most existing approaches are linear, which might be a good first-order approximation but might be limiting in practice to control a nonlinear plant.

A major criticism of the modularity hypothesis is that, for a given task or set of tasks, it is always possible to reduce dimensionality to a certain extent or to account for muscle patterns if a sufficient set of modules is extracted. As a consequence, the extracted modules may be inherently task-dependent and may reflect the biomechanical constraints imposed by the human body (especially for neighboring muscles that control the same joint) [90, 160]. Moreover, if optimality drives human trajectory formation and/or optimization guides muscle pattern design, it is clear that, in general, small task variations (e.g. changing a target's position) will only produce small muscle activity variations. Overall, this would lead to certain commonalities in muscle patterns, which could always be isolated by machine learning or statistical techniques. Nevertheless, similarly to optimal control, generalization should be tested and special care should be taken with regard to the conclusions that can be reached via such EMG-driven analyses. In other words, while describing the organization of muscle patterns is valuable to provide synthetic views of how ensemble muscle patterns are structured, relating this structure to underlying active neural mechanisms is tricky. In practice, the existence of circumstantial evidence, as described above, combined with the possibility that any deviations from the theory can be attributed to either a coarse model of modularity or alternative neural pathways/mechanisms (e.g. allowing individual muscle control) make the modularity hypothesis hard to falsify [155].

Consequently, although the muscle synergy hypothesis is popular in human motor control, it remains rightly debated. At the core of the debate is whether EMG-based synergies are just a descriptive low-dimensional representation of expectedly well-structured motor outputs or whether they have a real neural basis. To better address this important point, we can hypothesize that such a representation in terms of synergies exists and examine what this would imply [35]. First, a control action achieving a given motor task is hypothetically built from the combined activation of a given set of synergies. Therefore, performing a new task would just require adjusting the way synergies are combined until a suitable one is found. If no combination of synergies allows to execute the task, it may be because it requires non-habitual muscle patterns that have not been experienced and stored before, thus new synergies may need to be learned[12]. This logical reasoning predicts that learning a task that is incompatible with a currently available set of synergies would be harder and take a longer time. d'Avella and collaborators nicely investigated the predictions of such a theory along these lines [9]. This approach of a) formulating predictions of the modularity hypothesis and b) designing experiments to critically (and, if possible, quantitatively) test them might provide more direct evidence either supporting or falsifying the theory even though the neural code is not directly accessed [2]. Studying behavior and its adaptation may actually represent a very valuable approach to motor neuroscience [89].

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[12] In particular, this would be compatible with the claim that muscle patterns are habitual rather than optimal [123].

4 From optimality to modularity and vice-versa

4.1 Optimality with modularity: theoretical ground

Optimality and modularity theories have often been treated separately yet some studies have attempted to combine the two concepts. Theoretical works are of particular interest among those studies. Interesting frameworks have been derived to understand if (and in what contexts) optimal controllers can be built from a limited set of elementary control actions (possibly optimal themselves). Most of the times, the mathematical analyses aiming at reconciling the two approaches were conducted from a control theory perspective. The link between optimality and modularity may be envisioned as follows (Eq. 3):

$$\begin{array}{rrcl} \text{Dynamics}: & \dot{x} & = & f(x,u) \\ \text{Optimality}: & J(u) & = & \int_0^T h(x,u,t)dt \\ \text{Modularity}: & u(x,t) & = & \sum_{i\in\mathcal{I}} a_i v_i(x,t) \end{array} \tag{3}$$

where u is the control action that can drive the system state x according to some dynamical constraints specified by $f(x,u)$ and which is built from certain motor primitives or basis modules $v_i(x,t)$[13], h is the infinitesimal cost whose integral should be minimized. The main open question concerns the existence of motor building blocks allowing to effectively control the system for a given set of tasks. The basis modules may constitute a finite set of mixed feedforward and/or feedback control actions [109, 3, 4]. Linearity is often central to modularity studies because of experimental findings showing linear summation properties of spinal force fields (see Section 3). Interestingly, it can also be thought as a first-order approximation which simplifies the mathematical derivations. Notably, it allowed researchers to obtain elegant results for a restricted class of problems such as linear or feedback linearizable systems with quadratic costs or control-affine stochastic systems with control-quadratic costs that lead to a linear Bellman equation under suitable assumptions about noise [109, 147, 146]. These works showed that new optimal controls may be constructed from linear combinations of a finite number of elementary optimal control actions. However, the problem

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[13] The basis modules $v_i(x,t)$ might be separated into spatial and temporal components $\sigma_i(t)w_i(x)$ such as in [103] or [95], and in a way which is reminiscent of the model proposed in [43]. In this case, spatial (state-dependent) modules, or muscle synergies, would be feedback-dependent as suggested in [118]. Analogously, this time-space separation is also apparent in the optimal control of finite-horizon LQR/LQG problems.

may be more complex when thinking of the human motor system as a whole because of the hierarchical nature of the neuromusculoskeletal system. Optimal control may indeed occur at different levels in the hierarchy (kinematic, dynamic, muscular or neural levels). It is, apropos, particularly remarkable how relatively simple optimal control models (e.g. minimum jerk model) capture the hand/joint kinematics although they neglect fine muscle properties (e.g. speed/length dependencies, concentric/eccentric contractions, slow/fast twitch muscle fibers etc.). Therefore, optimal control may conceivably apply in a low-dimensional space (task space or joint space) and lower level neuromuscular activity may subsequently conform to these higher level constraints already specifying the main spatiotemporal characteristics of the movement. In the spirit that approximate optimal motor commands are acceptable, a hierarchical control framework has been proposed [149], thereby providing a theoretical link between task parameters and motor synergies. Other approaches based on deep learning schemes have also been considered by acknowledging that an optimal control problem readily implements a (nonlinear) dimensionality reduction [10] which could lead to very effective movement representations in neural networks. Although defining motor building blocks in a compositional sense seems harder in those frameworks, they nevertheless nicely capture the idea of dimensionality reduction resulting from the concept of motor synergies and address important questions such as whether the monitoring of a restricted number of task variables can yield suitable coordination of the complex and nonlinear musculoskeletal plant.

4.2 Optimality versus modularity: paradox and causality

At first sight, the coupling between optimality and modularity may seem paradoxical as the constraints imposed by modularity might severely compromise optimality. If one assumes existence of synergies, two seemingly competing questions arise: (1) whether modularity constrains and shapes the type or degree of optimality that can be attained in higher-level variables (i.e. end-effectors or joints) and (2) whether optimality naturally leads to an apparent modularity at the lower level (i.e. neuromuscular). On one hand, a number of studies have analyzed the extent to which motor synergies may arise from optimal control. Since optimal control and/or optimization may give *de facto* a structure to the associated muscle patterns according to variations of the task demand, it can reasonably be expected that empirical motor synergies are just a byproduct of optimality conditions. Accordingly, motor synergies have been shown to emerge from optimal feedback control theory [148]. Furthermore, numerical studies have provided interesting insights regarding what kind of time-varying synergies would result from optimal control policies applied to planar reaching movements [25]. On the other hand, the authors reversed the process to verify what kind of arm trajectories could be achieved via the extracted time-varying synergies [25]. Interestingly, these numerical experiments demonstrated that task constraints (reaching to targets in a plane) together with optimality objectives could be fulfilled with a small number of hypothetical motor synergies. In the same vein, other authors have investigated a similar issue for different tasks and subsystems. Computational studies based on musculoskeletal modeling showed that a simplified construction of motor commands via modularity (either experimentally inferred using dimensionality reduction techniques or synthesized) could significantly affect optimality and lead to sub-optimal solutions in terms of effort costs during balance control [98] and to a limited ability to minimize energy but also to tune endpoint stiffness during an isometric upper-limb task [77]. It is worth mentioning that these studies relied on optimization rather than optimal control[14]. Other studies involving optimization techniques are also relevant here as they implemented the whole loop mentioned in Eq. 2 using EMG-driven virtual biomechanics [123, 40]. They could quantify how muscle synergies affected energy consumption and aiming errors in task-space. Numerical studies about modularity have also been conducted in robotics [109, 3, 137], in which errors in task-space were also evaluated. Overall, it may be concluded that the nature of the motor building blocks (feedback/feedforward) and the type of system and cost (linear-quadratic) is critical for effective and efficient motor control using modularity. As suggested by Neptune's studies using complex musculoskeletal models, muscle synergies (as extracted in EMG-based studies) may serve as a rough starting point for motor planning, which should then be refined through spinal and transcortical reflex pathways. As such, modularity may be favorable to reduce the computational burden of behaving rigorously in an optimal way at the price of sacrificing some optimality.

Besides the question of the neural origin of muscle synergies, which is crucial to motor neuroscience (as already discussed) and proved difficult to answer [70], the chicken-and-egg situation of modularity versus optimality is of interest. Assuming that both optimality and modularity are present in the sensorimotor system, which one drives the other remains unclear. It is likely that de-

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[14] Optimization and optimal control should not be confused although they may be related when one comes to numerical resolution of optimal control problems. The former only deals with a standard function while the latter deals with a functional, i.e. a function of a function.

velopmental studies could help disentangling how optimality and modularity emerge during growth [46] and whether one constrained the other or vice-versa. It is possible that, if stored muscle synergies are hardwired in the CNS, they would limit the kind of optimality that can be attained (unless perhaps they have been precisely shaped to match the desired optimality conditions); this would suggest that motor development is the parallel process of finding both efficient limb trajectories and efficient representations of them, which would be partly dictated by evolutionary/homeostatic and neuroanatomical/biomechanical constraints. On a timescale of hours, it seems that motor learning is constrained by the existence of precoded/habitual solutions and that humans are not very good at finding global optima for new (never experienced) tasks [55, 123, 100]. It must however be noted here that recent studies actually undermine this temporal limitation [31, 131, 61, 50]. Yet, at least over long learning periods, a process of re-optimization may occur [81], possibly requiring the creation of new primitives, which would take a long time to acquire [35, 9].

5 Conclusion and perspectives

Hierarchical motor control, in particular grounded on the concepts of optimality and modularity, is an appealing theory to explain the formation of muscle patterns from task parameters. Although it is unclear whether global optimality, in a strict sense is the immediate and primary goal of the human motor system when adapting to a new task on a short-time scale, daily life behaviors (such as reaching for a cup of coffee) undeniably display optimal-like signatures. Inverse optimal control offers a normative framework to formalize action selection and give the rationale for choosing one limb trajectory over another. The difficulties that the CNS faces in order to discover a global optimum when coping with a certain new motor task may be related to the way action planning is implemented; what is stored or learned versus what is specified on-the-fly. How the CNS seamlessly generates adequate motor commands in a fraction of a second might rely on modularity, i.e. the storage of muscle synergies that can be recalled and combined in a task-dependent manner to build genuine motor signals. If muscle activity conforms in some sense to higher-level optimality principles, it is not surprising that low-dimensional structures can be found in EMG-based studies. Classical empirical studies of muscle activity are undoubtedly useful for providing ensemble descriptions of muscle patterns, yet their implementation in neural networks (third Marr's level of analysis) remains putative despite remarkable efforts [68, 112]. At the same time we remain quite ignorant of what the (neural) nature of a motor building block (should it be called primitive, module or muscle synergy) is. The nature of the modules extracted from matrix factorization techniques applied to EMG data is often guided by the limitations of the method itself. For instance, NMF-based methods will only give rise to feedforward synergies. If synergies are posture-dependent as suggested in [118], this would mean that more advanced machine learning methods should be used. Towards an anthropomorphic motion factory and effective applications in robotics, we believe that there is still place for the proposal of new frameworks. The development of such a novel framework requires interdisciplinarity and will require advanced musculoskeletal models, neurophysiological data, psychophysical experiments and mathematics.

A generic framework capturing the essence of both optimality and modularity could be useful to advance our understanding of human movement control. Importantly, the mathematical formalism should be suited to port the main findings to robotics and engineering in order to improve the production of efficient movements in artificial systems in a rapid, robust and adaptive way. The idea of hierarchy being central and ubiquitous in motor control, it may be the cornerstone of such a framework. Based on previous experimental and computational results, a model relying on a cascade of optimal control problems could be envisioned. The problems could be solved recursively by the CNS: each problem could integrate a nominal trajectory coming from the previous problem and serving as a reference trajectory to the current level. An example of such a hierarchical framework connecting task-level to muscle level via skeleton level, could be as follows:

$$\begin{array}{cc}
x_{ref}(\cdot)\ \text{built from task} & \text{(task level)} \\
\downarrow x_{ref} & \\
\min_{\tau(\cdot)} \int_0^T [h(q,\tau,t) + c(q,x_{ref})]dt & \text{(skeleton level)} \\
\downarrow \tau_{ref} & \\
\min_{u(\cdot)} \int_0^T [H(\eta,u,t) + C(\eta,\tau_{ref})]dt & \text{(muscle level)} \\
\downarrow u_{ref}\ \text{(motoneurons)} &
\end{array} \tag{4}$$

In Eq. 4, x would be the position/orientation of the end-effector in Cartesian space, x_{ref} would be a reference trajectory (either a fixed target location, a geometric path such as a geodesic or a full trajectory potentially coming from a least action principle or imposed by the task with a metronome). At the skeleton level, q would be the joint angles, τ the joint torques. A reference torque profile could be predicted by optimal control principles. At the muscle level, η would be the muscle forces and u the motoneuron inputs. An optimization problem could even be solved at this level if the spatiotemporal characteristics of the torque profiles are already set up. The core ingredient would be that each level would lead to a reference trajectory that could be tracked in the subsequent level. The tracking is implemented via the tracking costs $c(q, x_{ref})$ and $C(\eta, \tau_{ref})$, for instance by choosing at the skeleton level $c(q, x_{ref}) = c(\|\varphi(q) - x_{ref}\|)$ where $x = \varphi(q)$ is the forward kinematics (and something similar at the muscle level). Because of the existence of reference trajectories, the resolution of each optimal control problem may turn out to be faster and simplified (e.g. linearization or reduction of the search space). In this framework, we might insert other levels, introduce stochastic models and so on, but the structure is rich enough to incorporate a number of practical motor control problems.

Synergies may contribute to resolve each of these problems more efficiently, which could by the way explain why researchers have talked about modularity at various levels (kinematic, dynamic, muscular or even neural). Such a cascade of optimal control problems could be able to explain several experimental observations but this would remain to be investigated. This formulation may be reminiscent of an alternative formulation of hierarchical optimal control which has been proposed recently and in which a first optimal control problem giving rise to an infinity of solutions is solved before a subsequent optimal control problem is solved within the subspace of the optimal solutions of the previous level and so on [122, 62]. The present hierarchical framework may be flexible although it relies on the weighting of complementary objectives, thereby contrasting with the concept of a stack of tasks. Furthermore, this framework could account for the observation of kinematic persistence observed in some motor tasks, especially when controlling a visual cursor on a screen or adapting to microgravity, which may reflect the influence of task-space high-level goals in certain contexts [59, 33, 100, 61].

Appendix

In this appendix, we provide a tutorial example illustrating the concepts of optimality and modularity that we discussed in the main text.

To this aim, we consider a simple controlled pendulum whose dynamic equation is

$$\ddot{\theta} = -\theta + u$$

where θ is the angular position and u the input torque. This may represent a simplified human arm model in the gravity field (with normalized anthropometry and small angle assumption).

We define an optimal control problem as follows: find the controller that drives the system from a given state $x_0 = (\theta_0, \dot{\theta}_0)$ to a final state $x_f = (\theta_f, \dot{\theta}_f)$ in time T and minimizes the cost function

$$C(u) = \int_0^T [u^2 + \theta^2 + \dot{\theta}^2] dt.$$

This is a linear-quadratic (LQ) problem of the form $\dot{x} = Ax + Bu$ (linear **dynamics**) and $C(u) = \int_0^T u^T Ru + x^T Qx dt$ (quadratic **optimality** criterion) where the matrices are identified as follows:

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, R = 1, Q = I.$$

It can be shown that the optimal control can be written formally as (see [51] for mathematical proofs):

$$u(t) = -B^T P_+ \mathrm{e}^{A_+ t} p - B^T P_- \mathrm{e}^{A_-(t-T)} q \tag{5}$$

where P_+ and P_- are the maximal and minimal solutions of the associated Riccati equation $PA + A^T P - PBB^T P + Q = 0$. The matrices A_+ and A_- are defined as $A_+ = A - BB^T P_+$ and $A_- = A - BB^T P_-$, and p and q are some vectors depending on the initial/final states and on movement duration T. Importantly, the matrices $P_\pm$ and $A_\pm$ just depend on the optimal control problem specification, i.e. the matrices $A, B, R,$ and Q.

The optimal control $u(t)$ can thus be written as a function of the eigenvalues of $A_\pm$. Simple computations show that the 4 eigenvalues are of the form $\pm\alpha \pm \mathrm{i}\beta$ with $\alpha = \sqrt{(2\sqrt{2} - 1)}/2$ and $\beta = \sqrt{(2\sqrt{2} + 1)}/2$.

Therefore, the optimal control can be rewritten as the following linear combination:

$$u(t) = a_1 \mathrm{e}^{-\alpha t}\cos(\beta t) + a_2 \mathrm{e}^{\alpha t}\cos(\beta t) + a_3 \mathrm{e}^{-\alpha t}\sin(\beta t) + a_4 \mathrm{e}^{\alpha t}\sin(\beta t),$$

where the coefficients $(a_i)_{1\leq i\leq 4}$ have to be adjusted depending on the constraints $x(0) = (\theta_0, \dot{\theta}_0)$ and $x(T) = (\theta_f, \dot{\theta}_f)$.

In summary, this simple example illustrates that for this system all optimal motor commands can be decomposed as follows:

$$u(t) = \sum_{i=1}^{4} a_i v_i(t)$$

where the functions $v_i(t)$ are time-varying primitives that are invariants of the problem and can thus be stored once for all (**modularity**). In contrast, the activation coefficients a_i must be set for each single movement in order to start/end in the adequate states and times. Note the similarity between the present equations and Equation 3 in the main text. Storing invariant building blocks v_i and adjusting activation coefficients a_i in order to produce controllers that allow task achievement in an optimal fashion are the core concepts discussed in the present Chapter. It is worth noting that the same conclusion could actually be drawn for any well-defined LQ problem given the general form of Equation 5.

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