INTRODUCTION

M2 AI — SIGNAL PROCESSING

CONTENT

- Organisation
  - Short introduction for each sequence (30 min)
  - Working with Matlab or Python (your choice. Even Julia)
  - Groups of two students
  - Free organization: refer to theory by yourself if needed, ask questions (during the class or by mail)
- What is expected
  - Theory must be understood
  - Practical work must be done
  - A jupyter notebook (Python) or Matlab Publish (Matlab) to present your work
- Ressources
  - My website: http://hebergement.universite-paris-saclay.fr/mkowalski
  - Mail: matthieu.kowalski@universite-paris-saclay.fr ; matthieu.kowalski@inria.fr
  - Numerical tours: https://www.numerical-tours.com

SIGNAL: INTUITIVE DEFINITION

- A signal is a physical representation that carries “information” from a source to a recipient.
- It is a quantity physically measurable by a sensor, which can vary over time and/or space

EXAMPLE: MUSICAL (SOUND) SIGNAL

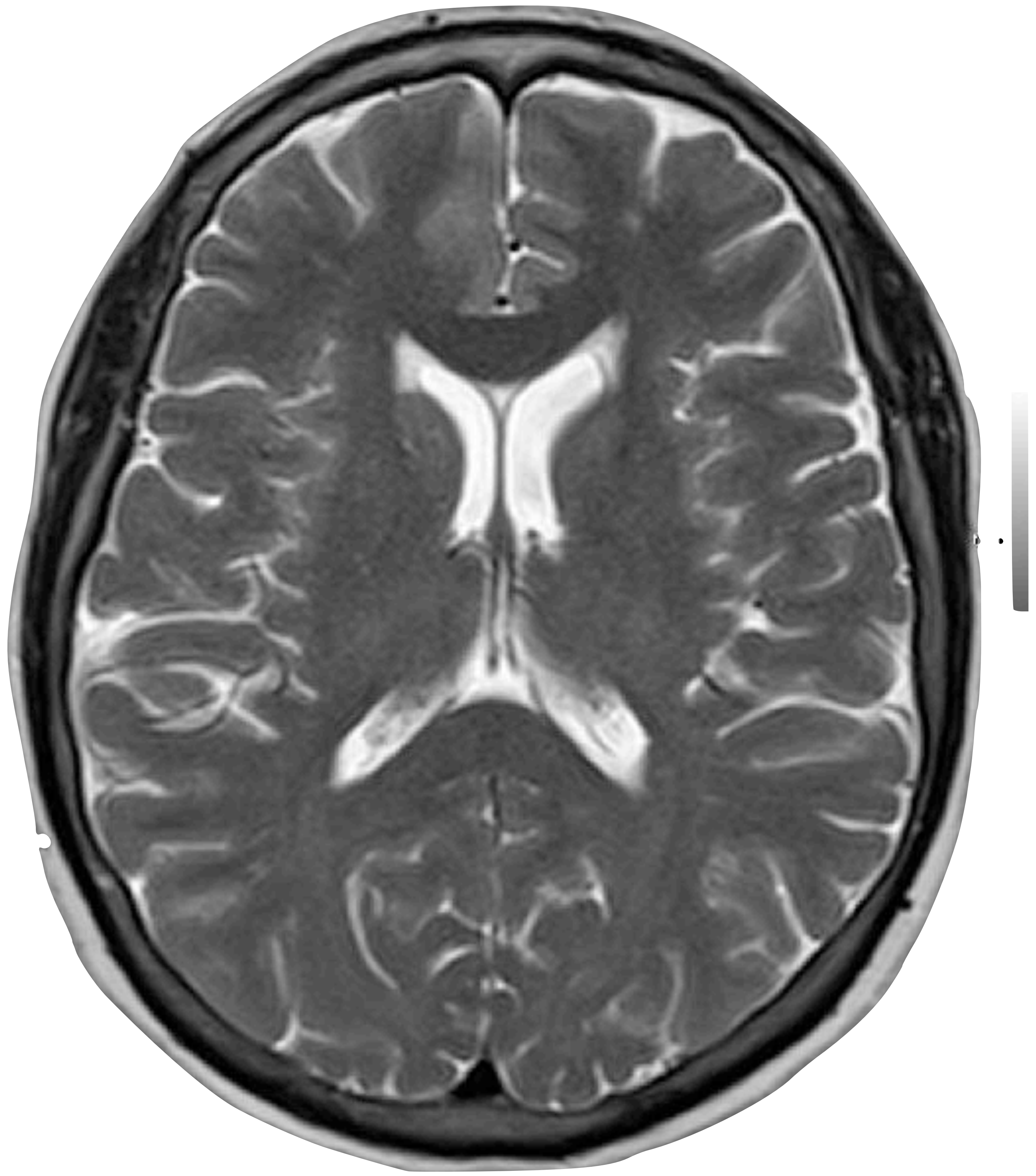
- Measured quantity: variation of sound pressure

EXAMPLE: PHOTOGRAPHY

- Measured quantity: photoelectric effect

EXAMPLE: IRM

- Measured quantity: magnetic field



EXAMPLE: ECHOGRAPH

- Physical quantity measured: Doppler effect by ultrasound

GOALS OF SIGNAL PROCESSING

- Model
- Analyze
- Restore
- Transmit
- Compress
- Solve inverse problems
- …

DETERMINISTIC VS RANDOM

- **Deterministic model:** signals that can be predicted "certainly" using simple descriptors (its function, its frequencies, etc.)
  Example: recording of a song, photography, etc.
  Tools: Fourier, Wavelets, Time-Frequency analysis, Ceptral analysis (Hilbert/Harmonic analysis)
- **Random model:** signals from "stochastic processes": each "realization" shares common quantities, but each signal is different.
  Example: background noise
  Tools: Probabilty, statistics, Estimation, Bayesian inference
- The two models are complementary
  Example: noisy signal

DETERMINISTIC SIGNALS: MATHEMATICAL DEFINITION

- An **analog** signal is a function of a **continuous variable**, usually "continuous" time. Let $s(t)$ be an analog temporal signal

$$\begin{aligned} s \ : \mathbb{R} &\to \mathbb{R} \\ t &\mapsto s(t) \end{aligned}$$

- A **numerical** signal is a function of a **discrete variable**, generally “discrete” time, that is to say, a mathematical sequence. Let $s[t]$ be a numerical time signal

$$\begin{aligned} s \ : \mathbb{Z} &\to \mathbb{R} \\ t &\mapsto s[t] = s_t \end{aligned}$$

FROM ANALOG TO DIGITAL

- Two steps
  1. Sampling
  2. Quantization
- Sampling Theorem:

Let $x(t)$ an **analog, band-limited signal**, i.e. with cutoff frequency ν_0. Then, $x(t)$ can be **reconstructed perfectly** from the samples $x(t_n)$ collected at the moment

$$t_n = \frac{n}{2\nu_0} = \frac{n}{\nu_e}$$

- $\nu_e = 2\nu_0$ is called the **sampling frequency.**
- The reconstruction formula is given by

$$x(t) = \sum_{n=-\infty}^{+\infty} x(t_n) \mathrm{sinc}\left(\nu_e(t - t_n)\right)$$

FROM ANALOG TO DIGITAL

- Two steps

  1. Sampling

  2. Quantization

- Quantization

  Each values $x(t_n)$ must be mapped from a real value (infinite precision) to a decimal value (with finite precision).

DEFINITIONS

A signal $s(t)$ is

- Causal iff $s(t) = 0 \quad \forall t < 0$
- Stable iff $\|s\|_1 < +\infty \left(\|s\|_1 = \sum_{k=-\infty}^{+\infty} |s[k]| \right)$
- Of finite energy iff $\|s\|_2^2 < +\infty \left(\|s\|_2^2 = \sum_{k=-\infty}^{+\infty} |s[k]|^2 \right)$
- Realisable iff $s(t)$ is stable and causal

SEQUENCES

1. Spectral analysis and ideal filtering Audio: guitar tuner Image: zoom in imaging
2. Real time Filtering Audio: Delay effect (IIR + FIR) Image: Segmentation
3. Random Signal Audio: noise spectrum estimation and vocoder
4. Time-frequency Audio: audio denoising by spectral substraction
5. Wavelets for images Image: image compression and denoising by thresholding
6. Introduction to inverse problems Debluring or Superresolution using TV or wavelets