SPECTRAL ANALYSIS

M2 AI — SIGNAL PROCESSING

USEFULLNESS

- Spectral analysis: frequency content of a function
- Think about musical notes!
- Measure the similarity (correlation, angle) between pure (complex) sine and a signal
- Sines are eigen signals of time-invariant linear systems (filters)
- The definition depends on the mathematical model

ONE FORMULA TO RULE THEM ALL

- Let $s(t)$ be a signal (continuous, discrete, periodic…). Let $\epsilon_\nu(t)$ be the (continuous, discrete, periodic…) **complex sine** at the frequency ν, then

$$\hat{s}(\nu) = \langle s(t), \epsilon_\nu(t) \rangle$$

And

$$s(t) = \langle \hat{s}(\nu), \overline{\epsilon_\nu(t)} \rangle$$

With the appropriate inner product !

ANALOG FOURIER TRANSFORM

- Fourier transform

$$\hat{s}(\nu) = \int_{-\infty}^{+\infty} s(t) e^{-i2\pi\nu t} \mathrm{d}t$$

- Inverse Fourier transform

$$s(t) = \int_{-\infty}^{+\infty} \hat{s}(\nu) e^{i2\pi\nu t} \mathrm{d}\nu$$

DIGITAL FOURIER TRANSFORM

- Let a discrete finite sequence $s = \{s[0], \ldots, s[N-1]\}$ (i.e., a digital signal).
- Fourier Transform:

$$\hat{s}[k] = \sum_{t=0}^{N-1} s[t] e^{-i\frac{2\pi kt}{N}}$$

- Inverse Fourier transform

$$s[t] = \frac{1}{N} \sum_{k=0}^{N-1} \hat{s}[k] e^{i\frac{2\pi kt}{N}}$$

- The signal $s[t]$ and its Fourier transform $\hat{s}[k]$ are considered as N-**periodic sequences!**

MAIN PROPERTIES

- Linearity
- Energy and inner product preservation (Plancherel-Parseval theorem)
- Translation in time becomes modulation in frequency
- Modulation in time becomes translation in frequency
- Time scale
- Derivation
- Hermitian symmetry for a real signal
- Convolution for digital signals of same size N, (using zero-padding if necessary)

$$\widehat{u * v}[k] = \hat{u}[k]\hat{v}[k]$$

the convolution is supposed to be **N-periodic!**

SPECTRUM

- Let s be a signal and $\hat{s}$ its Fourier transform. The (power) spectrum of s is the (squared) modulus of $\hat{s}$:

$$\text{(Magnitude)Spectrum}(s) = \left\{ \, |\, \hat{s}[k] \,| \, \right\} \qquad \text{PowerSpectrum}(s) = \left\{ \, |\, \hat{s}[k] \,|^2 \, \right\}$$

- If s is real-valued, then the spectrum is symmetrical with respect to the frequency 0

TO DO: GUITAR TUNER

- Data:
  - single notes in wav files
  - Guitar notes and corresponding frequencies:
    - **E1:** 329.63 Hz ; **B2:** 246.94 Hz ; **G3:** 196.00 Hz ; **D4:** 146.83 Hz ; **A5:** 110.00 Hz ; **E6:** 82.41 Hz
  - Values of 1 cent for accuracy of each string: 0.15, 0.15, 0.15, 0.08, 0.08, 0.04 (see: http://zerocapcable.com/?page_id=225 and https://www.engineeringtoolbox.com/octave-bands-frequency-limits-d_1602.html)
- Goal

For each wav files:

  - Perform a spectral analysis
  - Automatically determine the played note with the accuracy in cent

TO DO: ZOOM IN IMAGES

- Data:
    - Any image you want
- Goal:

For a given image:

- Perform a spectral analysis
- Resize the image to get a smaller image of half size
    - in the space domain by subsampling
    - by resizing in the Fourier domain
- Resize the image to get a bigger image of double size
    - by sinc interpolation (zero padding in the Fourier domain)
    - by linear interpolation in the space domain