SPECTRAL ANALYSIS

M2 AI — SIGNAL PROCESSING



USEFULLNESS

- Spectral analysis: frequency content of a function
- Think about musical notes!
- Measure the similarity (correlation, angle) between pure (complex) sine and a signal
- Sines are eigen signals of time-invariant linear systems (filters)
- The definition depends on the mathematical model



ONE FORMULA TO RULE THEM ALL

• Let s(t) be a signal (continuous, discrete, periodic...). Let $\epsilon_{\nu}(t)$ be the (continuous, discrete, periodic...) complex sine at the frequency ν , then

And

With the appropriate inner product !

 $\hat{s}(\nu) = \langle s(t), \epsilon_{\nu}(t) \rangle$

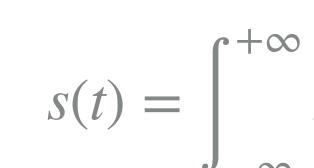
 $s(t) = \langle \hat{s}(\nu), \epsilon_{\nu}(t) \rangle$



ANALOG FOURIER TRANSFORM

Fourier transform

Inverse Fourier transform



$$\hat{s}(\nu) = \int_{-\infty}^{+\infty} s(t)e^{-i2\pi\nu t} dt$$

$$-\infty$$

 $\hat{s}(\nu)e^{i2\pi\nu t}\mathrm{d}\nu$

 $-\infty$



DIGITAL FOURIER TRANSFORM

- Let a discrete finite sequence $s = \{s[0], ..., s[N-1]\}$ (i.e., a digital signal).
- Fourier Transform:

 $\hat{s}[k] =$

Inverse Fourier transform

s[t] = -

The signal s[t] and its Fourier transform $\hat{s}[k]$ are considered as N-**periodic sequences!**

$$\sum_{t=0}^{N-1} s[t]e^{-i\frac{2\pi kt}{N}}$$

$$\frac{1}{N} \sum_{k=0}^{N-1} \hat{s}[k] e^{i\frac{2\pi kt}{N}}$$



MAIN PROPERTIES

- Linearity
- Energy and inner product preservation (Plancherel-Parseval theorem)
- Translation in time becomes modulation in frequency
- Modulation in time becomes translation in frequency
- Time scale
- Derivation
- Hermitian symmetry for a real signal
- Convolution for digital signals of same size N, (using zero-padding if necessary)

the convolution is supposed to be **N-periodic!**

 $u * v[k] = \hat{u}[k]\hat{v}[k]$



SPECTRUM

of \hat{s} :

(Magnitude)Spectrum(s) = { | $\hat{s}[k$

▶ If *s* is real-valued, then the spectrum is symmetrical with respect to the frequency 0

• Let s be a signal and \hat{s} its Fourier transform. The (power) spectrum of s is the (squared) modulus

$$k]| \} \quad \text{PowerSpectrum}(s) = \left\{ |\hat{s}[k]|^2 \right\}$$



TO DO: GUITAR TUNER

Data:

- single notes in wav files
- Guitar notes and corresponding frequencies:

E1: 329.63 Hz ; B2: 246.94 Hz ; G3: 196.00 Hz ; D4: 146.83 Hz ; A5: 110.00 Hz ; E6: 82.41 Hz

page_id=225 and https://www.engineeringtoolbox.com/octave-bands-frequency-limits-d_1602.html)

Goal

For each way files:

- Perform a spectral analysis
- Automatically determine the played note with the accuracy in cent

Values of 1 cent for accuracy of each string: 0.15, 0.15, 0.15, 0.08, 0.08, 0.04 (see: <u>http://zerocapcable.com/?</u>



M2 AI — SIGNAL PROCESSING — SPECTRAL ANALYSIS

TO DO: ZOOM IN IMAGES

Data:

Any image you want

• Goal:

For a given image:

- Perform a spectral analysis
- Resize the image to get a smaller image of half size
 - in the space domain by subsampling
 - by resizing in the Fourier domain
- Resize the image to get a bigger image of double size
 - by sinc interpolation (zero padding in the Fourier domain)
 - by linear interpolation in the space domain

