# FILTERING

## M2 AI — SIGNAL PROCESSING



## **SIGNAL AND SYSTEMS**

- A signal is recorded, and distorted, by a sensor
- A signal is (almost) always linked to the notion of "system"
- signal after applying a function to the input signal

$$x(t)$$
  $\longrightarrow$  S -

S is a "functional" that applies to a signal, and returns another signal

System: Functional block that reacts to an input excitation signal and produces a response



## FILTERS: LINEAR SYSTEMS INVARIANT IN TIME

- A filter is a linear system that is invariant over time
- Let S be a filter with an impulse response h[t]. Then

y[t] =

- For finite sequences (digital signals) of size N, 2 possibilities
  - Zeros-padding: h[t] = 0 and  $x[t] = 0 \forall t \notin \{0, \dots, N-1\}$

$$= S\{x\}[t]$$
  
=  $(h * x)[t]$   
=  $\sum_{k=-\infty}^{+\infty} h[t - k]x[k]$ 

• Circular convolution: h and x are supposed to be periodic (different results if the period is chosen to be N or 2N!)



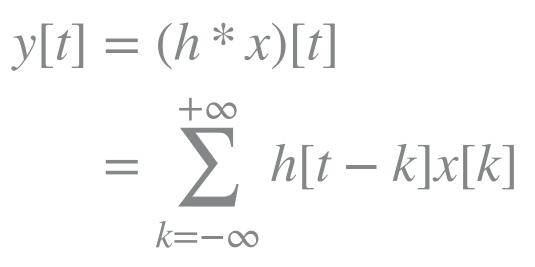


## FILTERING IN THE FOURIER DOMAIN

Let a filter with an impulse response h :

then (if the Fourier transform exists):

- done after zeros-padding !
- Filtering a signal acts directly on the spectrum



 $\hat{y}[\nu] = \hat{h}[\nu]\hat{x}[\nu]$ 

• For finite sequences (digital signals), the underlying convolution is circular: Fourier transform must be





## **IDEAL FILTERS AND REALIZABLE FILTERS**

- pass and Band cut filter.
- A filter is realizable iff its impulse response is stable and causal
- Ideal filters are not realizable (not causal)
- Two kind of realizable numerical filters:



, Infinite Impulse Response (IIR) filters: y[t] =

Ideal filter cut some frequencies while other are preserved. 4 types of filters: Low pass, High pass, Band

$$\sum_{n=0}^{K-1} h[n]x[t-n]$$

$$\sum_{n=0}^{+\infty} h[n]x[t-n] = \sum_{n=0}^{M-1} b[n]x[t-n] - \sum_{n=1}^{N-1} a[n]y[t-n]$$





## **DIGITAL FILTERS**

- Ideal filters can be implemented in the frequency domain (but not in real time)
- FIR filters are stable, but need a lot of coefficients
- IIR filters can be unstable, but accurate.
- Classical IIR filters: Butterworth, Tchebychev I & II, Elliptical



## **TO DO: FIR DELAY EFFECT**

#### Data:

Any (short) sound file

#### • Goal:

The FIR filter for delay effect can be implement thanks to the following input-output equation (e[t] is the input and s[t] is the output):

Where  $\alpha > 0$  is the attenuation factor and D is the time delay

- Implement the delay effect in the time domain
- Determine the impulse response of the filter (numerically)
- Provide the Frequency response of the filter (numerically)

```
s[t] = e[t] + \alpha e[t - D]
```



#### M2 AI — SIGNAL PROCESSING — FILTERING

## **TO DO: IIR DELAY EFFECT**

#### Data:

Any (short) sound file

#### • Goal:

The IIR filter for delay effect can be implement thanks to the following input-output equation (e[t] is the input and s[t] is the output):

Where  $\alpha > 0$  is the scaling factor,  $\beta > 0$  is the attenuation factor and D is the time delay

- Implement the delay effect in the time domain
- Determine the impulse response of the filter (numerically)
- Provide the Frequency response of the filter (numerically)
- Is this implementation always stable ?
- Discuss the parameters
- Compare with the FIR implementation

 $s[t] = \alpha e[t] + \beta s[t - D]$ 



#### M2 AI — SIGNAL PROCESSING — FILTERING

## **TO DO: IMAGE FILTERING**

Data:

Any image

#### ► Goal:

For a given image, implement the following filters and discuss their effects

- Gradient filter
- Sobel filter
- Averaging filter
- Gaussian filter

