FILTERING

M2 AI — SIGNAL PROCESSING

SIGNAL AND SYSTEMS

- A signal is recorded, and distorted, by a sensor
- A signal is (almost) always linked to the notion of "system"
- System: Functional block that reacts to an input excitation signal and produces a response signal after applying a function to the input signal

- S is a "functional" that applies to a signal, and returns another signal

FILTERS: LINEAR SYSTEMS INVARIANT IN TIME

- A filter is a linear system that is invariant over time
- Let S be a filter with an impulse response $h[t]$. Then

$$\begin{aligned} y[t] &= S\{x\}[t] \\ &= (h * x)[t] \\ &= \sum_{k=-\infty}^{+\infty} h[t-k]x[k] \end{aligned}$$

- For finite sequences (digital signals) of size N, 2 possibilities
  - Zeros-padding: $h[t] = 0$ and $x[t] = 0 \ \forall t \notin \{0,\ldots,N-1\}$
  - Circular convolution: h and x are supposed to be periodic (different results if the period is chosen to be N or $2N$!)

FILTERING IN THE FOURIER DOMAIN

- Let a filter with an impulse response h :

$$\begin{aligned} y[t] &= (h * x)[t] \\ &= \sum_{k=-\infty}^{+\infty} h[t-k]x[k] \end{aligned}$$

then (if the Fourier transform exists):

$$\hat{y}[\nu] = \hat{h}[\nu]\hat{x}[\nu]$$

- For finite sequences (digital signals), the underlying convolution is circular: Fourier transform must be done after zeros-padding !
- Filtering a signal acts directly on the spectrum

IDEAL FILTERS AND REALIZABLE FILTERS

- Ideal filter cut some frequencies while other are preserved. 4 types of filters: Low pass, High pass, Band pass and Band cut filter.
- A filter is realizable iff its impulse response is stable and causal
- Ideal filters are not realizable (not causal)
- Two kind of realizable numerical filters:
  - Finite Impulse Response (FIR) filters: $y[t] = \sum_{n=0}^{K-1} h[n]x[t-n]$
  - Infinite Impulse Response (IIR) filters: $y[t] = \sum_{n=0}^{+\infty} h[n]x[t-n] = \sum_{n=0}^{M-1} b[n]x[t-n] - \sum_{n=1}^{N-1} a[n]y[t-n]$

DIGITAL FILTERS

- Ideal filters can be implemented in the frequency domain (but not in real time)
- FIR filters are stable, but need a lot of coefficients
- IIR filters can be unstable, but accurate.
- Classical IIR filters: Butterworth, Tchebychev I & II, Elliptical

TO DO: FIR DELAY EFFECT

- Data:

Any (short) sound file

- Goal:

The FIR filter for delay effect can be implement thanks to the following input-output equation ($e[t]$ is the input and $s[t]$ is the output):

$$s[t] = e[t] + \alpha e[t - D]$$

Where $\alpha > 0$ is the attenuation factor and D is the time delay

  - Implement the delay effect in the time domain
  - Determine the impulse response of the filter (numerically)
  - Provide the Frequency response of the filter (numerically)

TO DO: IIR DELAY EFFECT

- Data:

Any (short) sound file

- Goal:

The IIR filter for delay effect can be implement thanks to the following input-output equation ($e[t]$ is the input and $s[t]$ is the output):

$$s[t] = \alpha e[t] + \beta s[t-D]$$

Where $\alpha > 0$ is the scaling factor, $\beta > 0$ is the attenuation factor and D is the time delay

- Implement the delay effect in the time domain
- Determine the impulse response of the filter (numerically)
- Provide the Frequency response of the filter (numerically)
- Is this implementation always stable ?
- Discuss the parameters
- Compare with the FIR implementation

TO DO: IMAGE FILTERING

- **Data:**

Any image

- **Goal:**

For a given image, implement the following filters and discuss their effects

  - Gradient filter
  - Sobel filter
  - Averaging filter
  - Gaussian filter