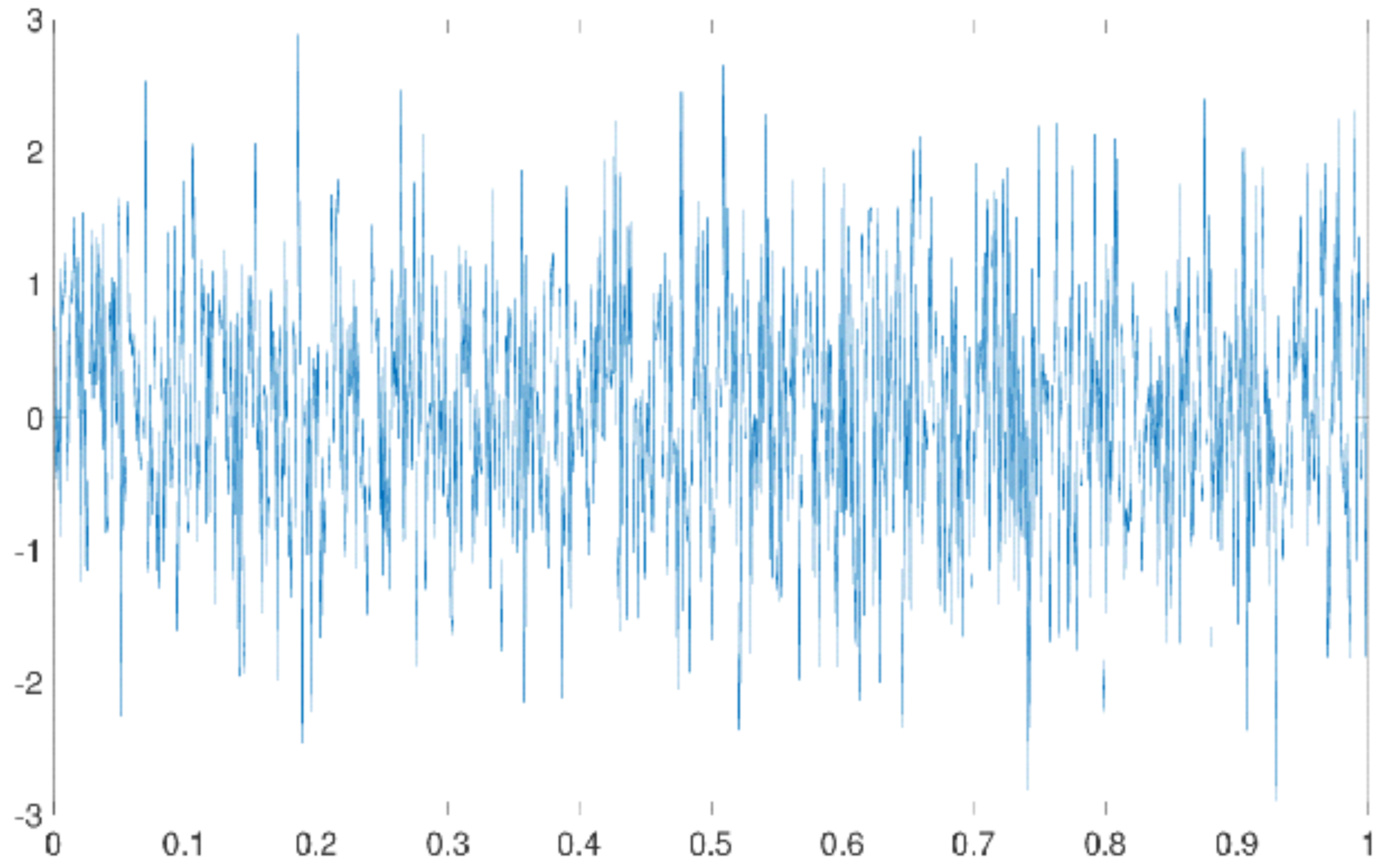
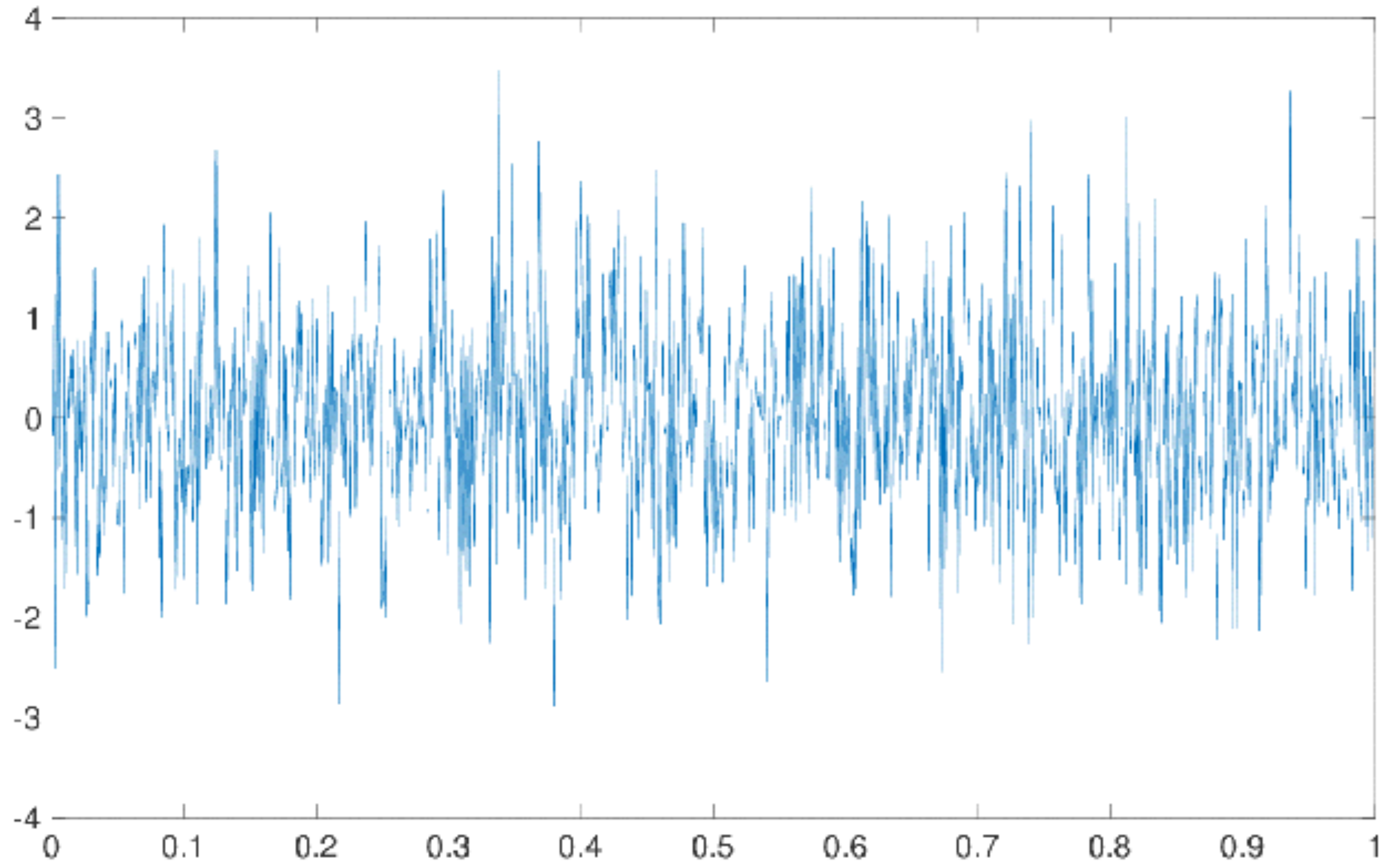
RANDOM SIGNALS

M2 AI — SIGNAL PROCESSING

INTRODUCTION

- Usefulness:
  - Noise modeling
  - Statistical estimation
  - Bayesian approach
- Different, but similar





DIGITAL RANDOM SIGNAL

- A digital random signal is a random process: $X[t]$
- For each t, $X[t]$ is a random variable
- In practice we observe one (partial) trajectory, that is one realization $\boldsymbol{x} = \{x[0], \ldots, x[N-1]\}$
- The random signal is, usually, assumed to be ergodic and stationary:
  - We can work with only one realization
  - The autocorrelation function $R_X[k-n] = \mathbb{E}\left\{X[k]X[n]\right\}$is deterministic
  - We can work with temporal statistics

SPECTRUM OF RANDOM SIGNAL

- Let X be a random signal
  - The Fourier transform of X is not meaningful
  - The spectrum of a random signal is the Fourier transform of its autocorrelation function:

$$S(\nu) = \hat{R}_x[t]$$

- Can be difficult to estimate in practice
- Two estimators:
  - Periodogram estimator (power spectrum of the observed trajectory x)
  - Welch estimator (average of short-time power spectrum)

NOISE

- Gaussian white noise
  - All the random variables $X[n]$ are i.i.d. from $\mathcal{N}(0,\sigma^2)$
  - Autocorrelation function: $R_x[t] = \sigma^2\delta[t]$
  - Spectrum: $S(\nu) = \sigma^2$
- A Gaussian colored noise is a filtered white noise

LINEAR DENOISING

- Let y be a noisy measure of a "clean" signal x corrupted by some additive noise n:

$$y = x + n$$

- Signal to Noise Ratio (SNR):

$$SNR(y\,|\,x) = 20\log\left(\frac{\|x\|}{\|y-x\|}\right)$$

- Goal: find the best (oracle) filter h such that $x_{est} = h * y$ is the best estimation of x
- Solution: Wiener filter, given in the frequency domain by

$$\hat{h}(\nu) = \frac{\mathbb{E}\{\,|\hat{x}(\nu)|^2\,\}}{\mathbb{E}\{\,|\hat{x}(\nu)|^2\,\} + \mathbb{E}\{\,|\hat{n}(\nu)|^2\,\}}$$

- More on the numerical tour !! (See the linear image denoising tour)

TO DO: NOISE SPECTRUM DENSITY ESTIMATION AND WIENER FILTERING

- Data
  - 3 noises
  - Audio file or image of your choice
- Todo
  - For each noise
    - Estimate the spectrum density by periodogram and Welsh method
    - Identify the color of the noise (white, pink, red, mixture of noises…)
  - With the image or audio file
    - Simulate a noisy version of the signal with various SNR (0dB, 5 dB, 10 dB, 15 dB, 20 dB), using a Gaussian white noise
    - Denoise the signal using the Wiener filter