TIME-SCALE (WAVELET) ANALYSIS

M2 AI — SIGNAL PROCESSING

CONTINUOUS WAVELET TRANSFORM

- Idea: be sensitive to irregularities instead of oscillations
- Let $\psi(t)$ be an admissible wavelet and its dilated and translated versions

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$

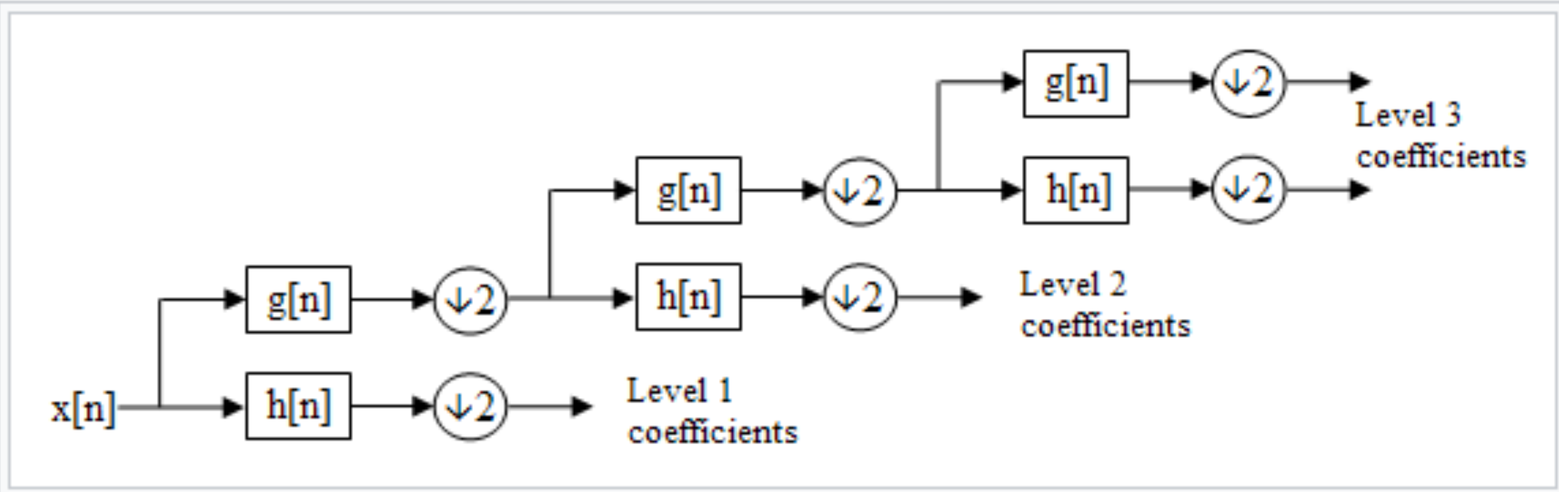
- The continuous wavelet transform is given by:

$$C_x(a,b) = \langle x(t), \psi_{a,b}(t)\rangle = \frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} x(t)\psi\left(\frac{t-b}{a}\right)\mathrm{d}t$$

- $|\,C_x(a,b)\,|$ is called the magnitude scalogram
- Properties
  - It is a time-scale transform
  - It is invertible: $x(t) = \frac{1}{c_\psi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} X(a,b)\psi\left(\frac{t-b}{a}\right)\frac{\mathrm{d}a\ \mathrm{d}b}{a^2}$
  - We have energy preservation

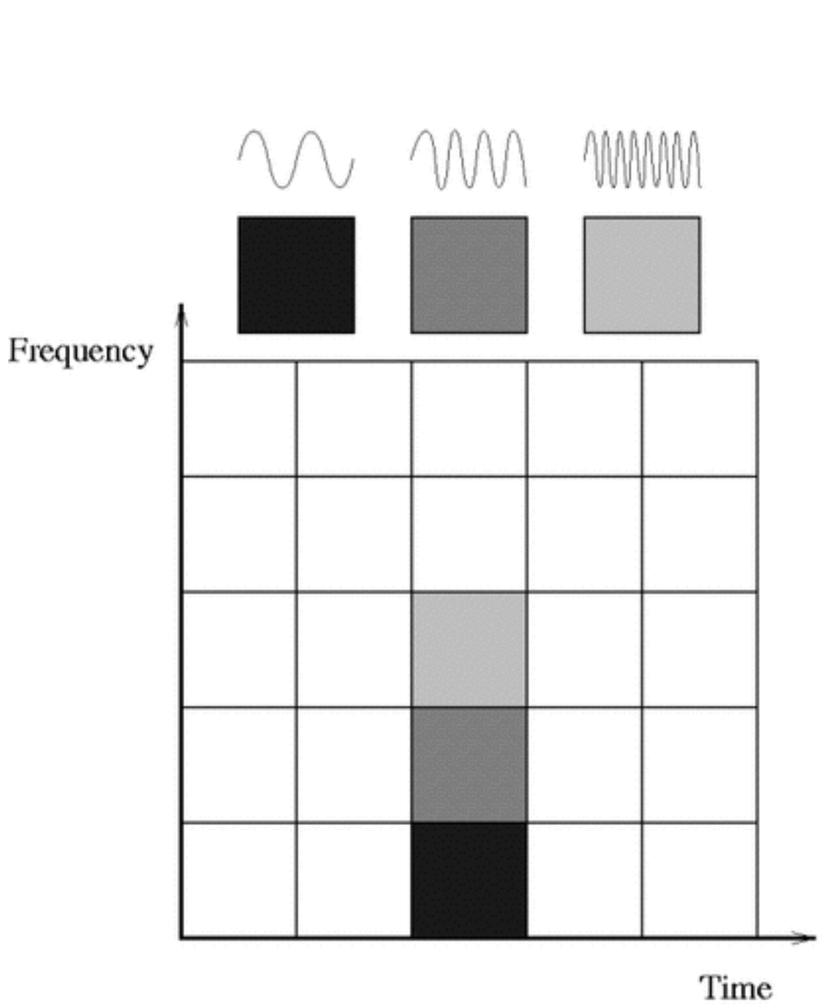
ORTHOGONAL WAVELET TRANSFORM

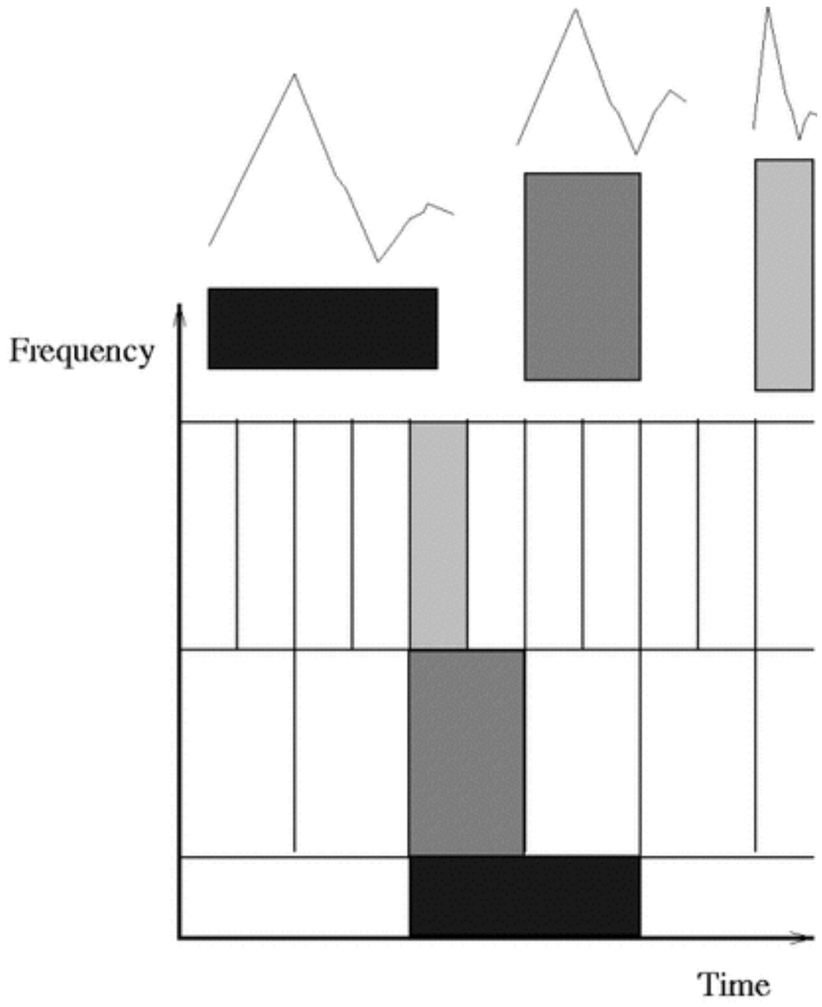
- Comes from “multiresolution” analysis on a dyadic scale
- In practice, a Fast (orthogonal) Wavelet Transform can be computed thanks to filter bank and subsampling
- The same filter bank can be used to reconstruct the signal from the wavelet coefficients
- A wavelet is then fully determined by two filters (which must fulfill certain conditions): a low pass filter g and a high pass filter h



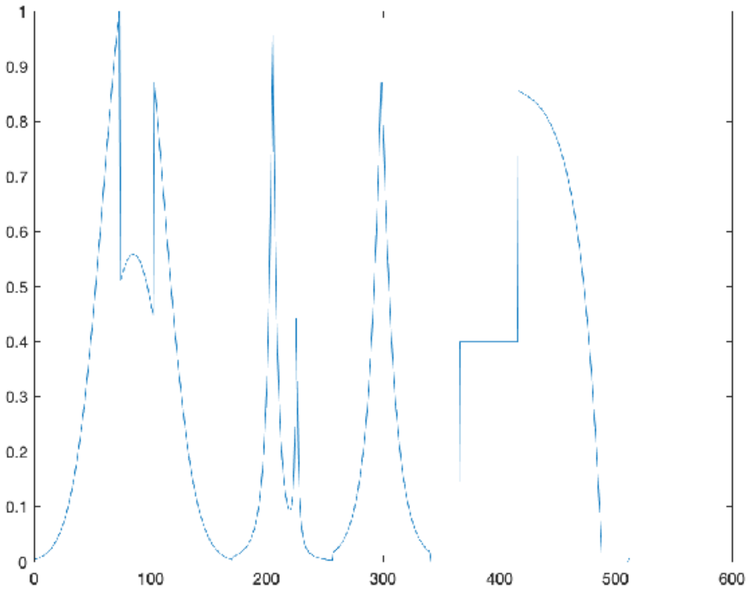
A 3 level filter bank

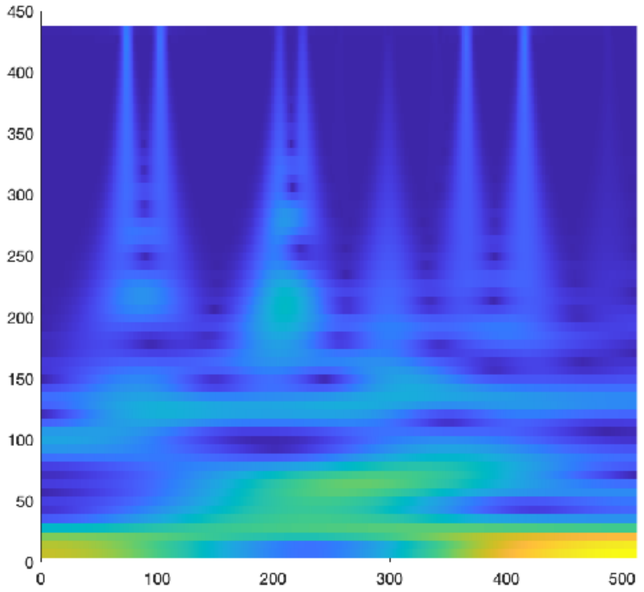
TIME-FREQUENCY TILING

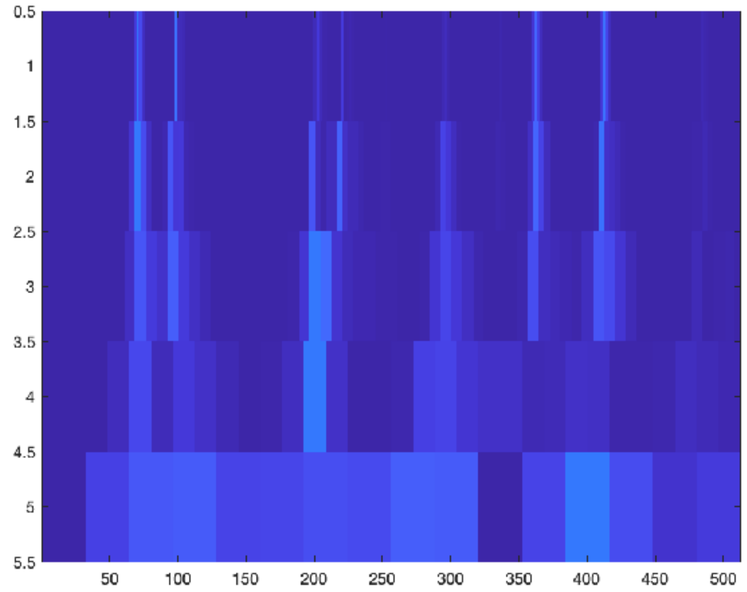




SCALOGRAM: CONTINUOUS VS ORTHOGONAL

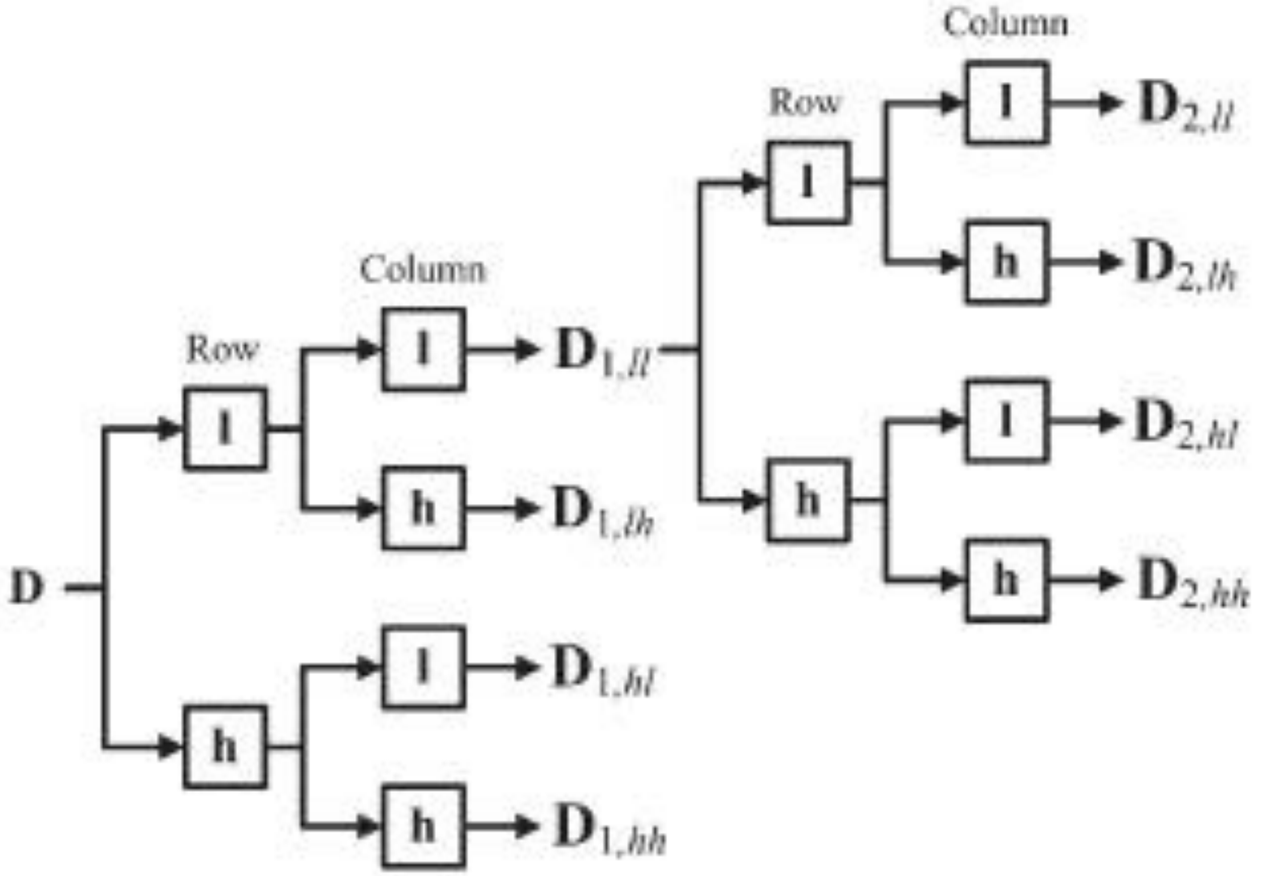




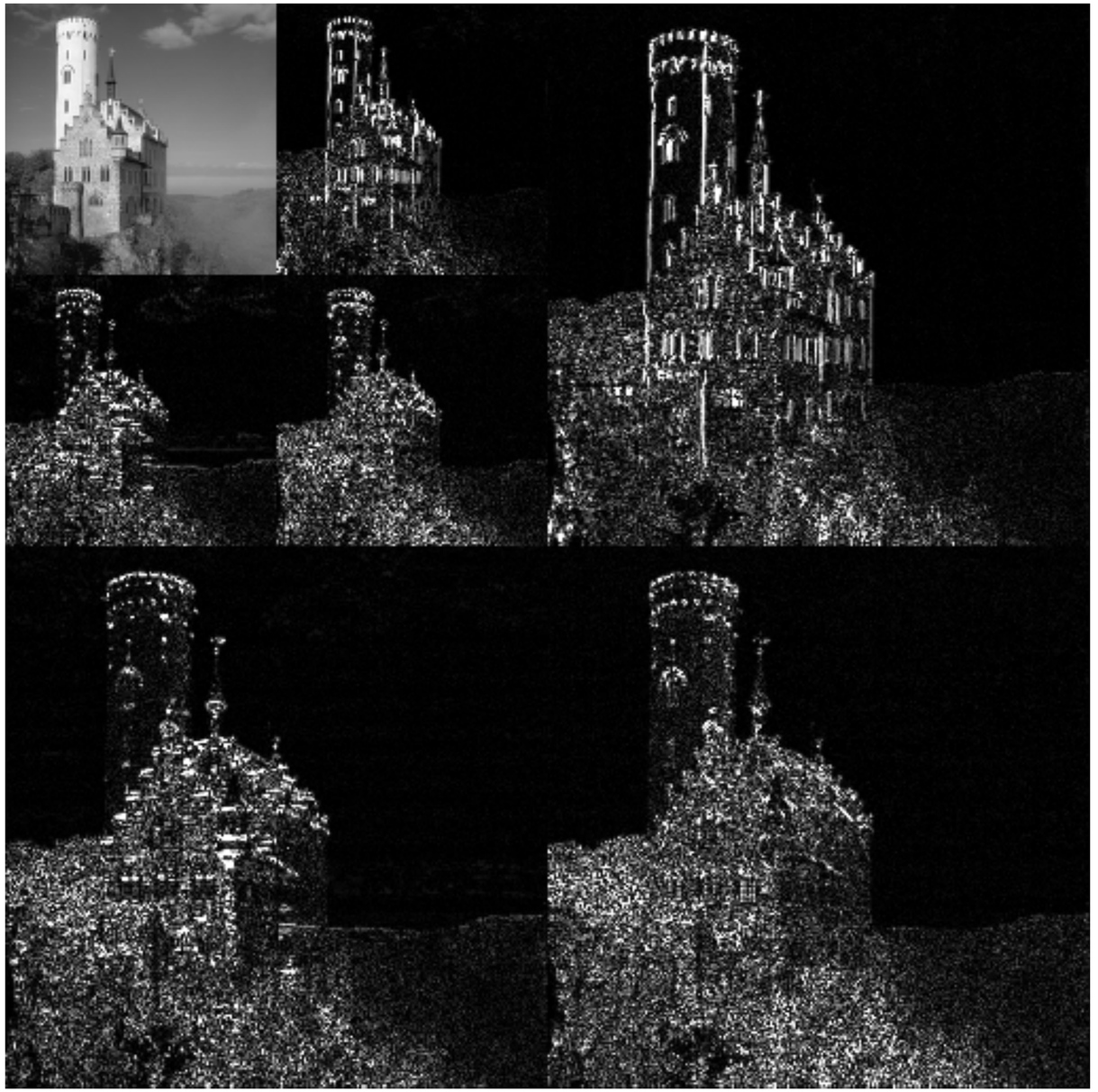


2D FWT

- For images, we need a 2D FWT
- It is constructed by using a separable filter bank (we first filter the row, then the column with the same filters)



2D FWT



TO DO: DENOISING IN THE WAVELET DOMAIN

- Data
  - Image you want
- Todo
  - Simulate a noisy version of the image using the noises at various SNR Level (0 db, 10 dB and 20 dB)
  - Denoise the image by performing hard thresholding and soft thresholding in the wavelet domain
  - Discuss the parameters (wavelet type, number of level, type of thresholding)
  - Denoise the image by implementing translation invariant wavelet transform (see the numerical tour on image wavelet denoising)