center of mass moves directly toward O), and thus we have only the "spin" contribution:

$$L_{0z} = -\tfrac{1}{3}mR^2\dot{\theta}. \qquad (14)$$

Setting $\tau_0 = \dot{\mathbf{L}}_0$, Eqs. (12)–(14) can be combined to produce Eq. (11) once again. It is reassuring to learn that with care we can arrive at the same differential equation whether we take O to be the center of mass (as in Crawford's *correct* solution), a point fixed in the surface of the table, or the sliding contact point.

In Crawford's article,[1] together with this one, three correct solutions to the sliding stick problem have been presented, one of them using a "dangerous" point and two using "safe" points (the center of mass and a point fixed in the surface of the table). We have made no attempt to present any solution using as a point about which moments are calculated one whose instantaneous acceleration is not zero but is directed toward or away from the center of mass. Although for such a point the simpler Eq. (3) is indeed correct, and though there are cases (an unweighed wheel or a homogeneous cylinder rolling on a surface) in which use of such a point provides a simple solution, in the interests of good pedagogy and clarity of thought we endorse the sentiments of Desloge[8] on this matter: "However the reader is advised to forget this special case. It is never necessary to use it, and the convenience that might be gained by retaining this case in one's repertoire is outweighed by the possibility of error it encourages."

A natural question may arise at this point. Why would anyone in their right mind want to solve a problem using a "dangerous" point, when it is easier to use the center of mass, or a point that is not accelerating? This question has at least three answers. First, in this particular problem, the solution of the problem by taking moments around the controversial point convinces us that Eq. (4) really works. Second, in general, it is always of pedagogical value to show that careful application of two or more methods yields the same (correct) results. Third, there are many cases (this being a particular case) in which the use of (4) gets rid of information we are not interested in. Suppose that we are just interested in finding the equation of motion of the object but are not concerned with the reactive forces. The use of (4) around a point such as O allows us to find what we want immediately. I claim that this would sometimes save some time but, above all... it is BEAUTIFUL.

ACKNOWLEDGMENT

I should like to thank Steve J. Eppell for a careful reading of an early draft of the manuscript.

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Rigid levitation and suspension of high-temperature superconductors by magnets

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A high-T_c superconductor floating freely above a magnet of low symmetry remains rigidly suspended in the air in almost any position and orientation as if stuck in an invisible heap of sand. This striking effect is due to pinning of the magnetic flux lines inside the superconductor and is often overlooked, since usually magnets with rotational symmetry are used for levitation. Magnets with rotational symmetry allow for nearly undamped orbiting and rotation of the superconductor about the magnet's symmetry axis. But even in this geometry, flux-line pinning can be seen, since it forces the orbiting superconductor to turn the same face toward the axis. Superconductors with sufficiently strong pinning may even be suspended below a magnet.

I. INTRODUCTION

Ever since the discovery of ceramic superconductors,[1] some of which remain superconducting above the temperature of liquid nitrogen ($T = 77$ K),[2] the levitation of a superconductor is one of the most impressive physical experiments. Figure 1 shows disks of the "high-temperature" superconductor $YBa_2Cu_3O_x$ ($x \approx 6.8$) with a transition temperature $T_c \approx 92$ K, submerged first in liquid nitrogen and then put above a strong magnet. This magnet generates

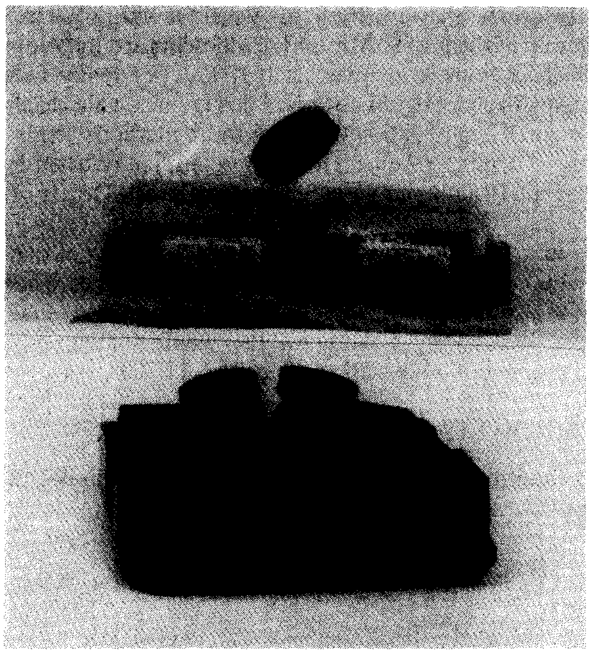

Fig. 1. Disks of the high-T_c oxide superconductor $YBa_2Cu_3O_{6.8}$ of diameter 12 mm floating freely above a magnet with strong lateral field gradients. The disks float motionless at any inclination and almost any position not too far from the magnet's center; even several disks can float simultaneously without touching each other. The magnet consists of four cobalt–samarium permanent magnets of size $2\times2\times1$ cm^3 with south pole up, mounted together with a central north pole on an iron plate. The magnet sits in a bowl with liquid nitrogen whose evaporating gas cools the superconductor. Note the white frost covering the magnet holder of copper.

an inhomogeneous magnetic field such that a disk is repelled and floats freely until it warms up to near its T_c. For a physicist who has worked with "usual" superconductors that require a complicated cryostat with liquid helium ($T = 4.2$ K), it is a fascinating experience now to be able to touch a superconductor and feel its magnetic force directly.

The levitation comes from the *expulsion of magnetic flux*

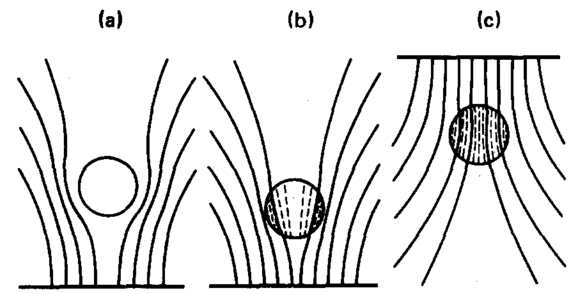

Fig. 2. Superconducting spheres floating freely in an inhomogeneous magnetic field. The solid lines denote magnetic field lines (schematic). (a) Ideal Meissner state, in which all flux is expelled from the interior of the levitating sphere (ideal diamagnetism). (b) Partial penetration of magnetic flux in the form of flux lines (dashed). The applied field exerts a pressure on the flux lines that is compensated by the repulsion between the flux lines and by the pinning forces exerted by material inhomogeneities. The sphere floats since part of the flux is expelled (nonideal diamagnetism). (c) A superconductor with stronger pinning in a decreasing field may trap sufficient flux to allow its free suspension below a magnet. The magnetic field lines in this case are drawn into the specimen, which thus behaves similarly to a ferromagnet.

from the superconductor as shown in Fig. 2(a). A permanent magnet or a piece of iron cannot float in a stable position above a magnet.[3] The observed stable levitation thus proves that the specimen (or part of it) is in the superconducting state or is at least ideally conducting.

A modified experiment in which the warm specimen is put on a magnet, and then cooled until it lifts off and floats is a more rigorous proof of superconductivity. This experiment displays the Meissner effect, i.e., that magnetic flux is expelled from the specimen when it is cooled below T_c.[4] In contrast to the superconductor, a normal conductor with zero resistivity would keep the magnetic field in it unchanged when it became ideally conducting in a magnetic field.

II. FRICTION DURING LEVITATION

Besides this free flotation, one observes another fascinating effect on the levitated new superconductors, an effect whose explanation is less obvious. With some magnets the levitated specimen does not rotate or swing about its equilibrium position, but rather remains *rigidly suspended* in the air as if it were stuck in an invisible medium that exerts friction on it. One may push the superconductor into almost any position not too far from the magnet's center and the superconductor will remain there motionless. Figure 1 demonstrates three different stable positions and orientations of similar disks levitating above the same magnet, two of them simultaneously.

This levitation with friction is unique. Other levitation mechanisms known in physics are free of friction,[3] e.g., levitation by gas jets, strong sound waves, or laser beams; by electric, magnetic, and radio-frequency fields; and also the often demonstrated levitation of a small magnet over a concave piece of superconducting lead (which is a type-I superconductor; see Sec. II).

Where does the strong frictional force come from? Obviously, the observed behavior requires that *energy is dissipated* when one forces the superconductor out of its position. However, if the magnetic flux were expelled completely from the specimen, there would be no way to dissipate energy. Only at high frequencies would electromagnetic radiation result from an oscillating magnet, and eddy current damping, which occurs, e.g., in a copper disk rotating in certain inhomogeneous fields, would exert a frictional force that is proportional to the velocity of the specimen. Apparently the observed friction in the ceramic superconductor stays finite at zero velocity similar to the dry friction between touching bodies.

III. TYPE-I AND TYPE-II SUPERCONDUCTORS

Since the magnetic field around the specimen is free of friction (it obeys Maxwell's equations, which in vacuum do not contain a dissipative term), the observed frictional force must originate from a process inside the superconductor. In the ideal Meissner state in which all magnetic flux is expelled from the bulk, there will be no friction: The expulsion is caused by shielding currents that flow in a thin surface layer of thickness $\lambda \approx 10^{-7}$ m, the penetration depth for weak magnetic fields.[4] Inside the superconductor the magnetic field generated by these surface currents exactly compensates the applied field. These shielding currents are completely loss-free "supercurrents." (The superconducting order parameter does not vanish near the surface.) Very little normal current flows in the penetration layer since the supercurrents tend to short circuit any electric field. Therefore, *superconductors in the Meissner state will float and oscillate with practically no damping.* Weak residual damping may result from eddy currents that the moving superconductor induces in the magnet or the magnet holder.

Nearly frictionless levitation, with orbits and oscillations about a unique equilibrium position, is indeed observed in so-called *type-I superconductors* (most pure metals, e.g., tin and lead). As opposed to these, most alloys and niobium and vanadium are *type-II superconductors.* Type-I superconductors have a Ginzburg–Landau parameter $\kappa < 0.7$ and type-II superconductors have $\kappa > 0.7$, where $\kappa = \lambda / \xi$ is the (almost temperature-independent) ratio of the magnetic penetration depth λ and the superconducting coherence length ξ. Approximate temperature dependences are $\lambda \sim \xi \sim (T_c - T)^{-1/2}$; at $T = 0$, to good approximations, λ equals the London penetration depth and ξ equals the extent of Cooper pairs.[4] The high-T_c oxide superconductors exhibit very large $\kappa \approx 100$ and are thus of extreme type-II.

The damping observed in levitated superconductors indicates that some of the magnetic flux has penetrated. Indeed, it was predicted by Abrikosov[5] that type-II superconductors may contain magnetic flux in association with tiny current vortices of radius λ, each carrying one quantum of flux $\phi_0 = h/2e = 2.07 \times 10^{-15}$ T m^2 (in SI units, h = Planck's constant and e = electronic charge). These flux lines were observed in 1967 by a decoration technique in an electron microscope[6,7] and they have been observed recently in the high-T_c superconductors.[8]

In ideal type-II superconductors vortices start to penetrate when the applied field B_a exceeds the lower critical field B_{c1} and arrange themselves in a regular triangular lattice with spacing $(2\phi_0/\sqrt{3B})^{1/2}$ where B is the average flux density or internal field. With increasing B_a the flux-line lattice becomes denser and vanishes when B_a (and B) reaches the upper critical field B_{c2}, above which the superconductor behaves as a normal conductor. Both B_{c1} and $B_{c2} \approx (2\kappa^2/\ln \kappa) B_{c1}$ are maximum at zero temperature and vanish when T is increased to T_c. At 77 K the new superconductors exhibit a B_{c1} of the order of 10^{-2} T (100 G), and B_{c2} extrapolates to over 100 T (10^6 G); see, e.g., Refs. 9 and 10. Both B_{c1} and B_{c2} are strongly anisotropic, i.e., they depend on the orientation of the applied field relative to the atomic lattice.

IV. PINNING OF FLUX LINES

When an applied current is driven through the flux-line lattice, this lattice may drift driven by a Lorentz force. Flux-line motion relative to the atomic lattice ("flux-flow") induces additional currents near each vortex that possess dipolelike stream lines and flow also through the normally conducting core of the vortex.[11,12] This motion dissipates energy (near the vortex center where the order-parameter vanishes) and induces a voltage drop along the specimen. The superconductor is then no longer ideally conducting. However, the *flux lines may be pinned by material inhomogeneities*, e.g., precipitates, grain boundaries of microcrystals, crystal lattice defects, a rough surface, etc. Current densities j smaller than a critical value j_c can, therefore, flow without loss. Since the current inside a type-II superconductor is always accompanied by a gradient in the flux density B (by a modified Ampere's law[13]), there exists a *critical gradient* in B above which the flux lines get unpinned just as sand starts to slide when a critical slope is reached.

The calculation of the critical current density j_c is a complicated statistical problem[13–18,7] into which enter not only the elementary pinning forces of the pins but also the elastic and plastic properties of the flux-line lattice.[19] Why the elasticity of the flux-line lattice enters may be seen from the following arguments. A *rigid* flux-line lattice of *infinite* size cannot be pinned by a random arrangement of pins, since then the forces on flux lines are uncorrelated and pinning forces of all orientations cancel mutually. (A rigid lattice of *finite* size may be pinned weakly by random fluctuations of the average force.) On the other hand, a *soft* flux-line lattice is pinned effectively, since now the flux lines adapt to the pins and most pinning forces will oppose the driving force exerted by an applied or induced current.

V. ORIGIN OF THE INTERNAL FRICTION

When the levitated superconductor is pushed to a position with different magnetic field, some flux tries to leave or enter the specimen in the form of flux lines. As a consequence, flux lines may have to move so as to maintain a critical gradient of B, which extends more or less deeply into the specimen. Moving flux lines have to be unpinned and thereby exert a frictional force on the atomic lattice; they behave like plucked strings embedded in a viscous medium.

The friction starts when, at any point of the specimen surface, the difference between the magnetic fields inside (B) and outside (B_a plus the stray field caused by the magnetization of the sample, see Sec. VII) the superconductor reaches a critical value. More precisely, the condition for the onset of flux flow is that the discontinuity in the component of the magnetic field parallel to the surface reaches a critical value $j_c \mu_0 \lambda$ (μ_0 is the permeability of the vacuum). At this threshold the shielding current density, decreasing exponentially into the interior, has a maximum value of j_c.[4,13] When the field discontinuity is smaller everywhere on the surface, then almost no energy is dissipated. A weak residual dissipation may result from "flux creep" caused by occasionally jumping (creeping or thermally activated) flux lines.

Oscillations of the specimen with very small amplitude are thus only weakly damped. With increasing vibrational amplitude, damping of a hysteretic type sets in, comparable with the dry friction between vibrating elastic bodies. A similar amplitude-dependent damping has been observed in superconducting reeds made of amorphous alloys[20] or of oxide superconductors.[10] When a reed clamped at one side is cooled below its critical temperature T_c, the attenuation of its flexural vibrations may increase by four orders of magnitude due to the unpinning of flux lines.

VI. GEOMETRIES WITH WEAK DAMPING

There are situations in which the damping of a floating superconductor is weak or even vanishes for certain types of motion because the shielding currents remain below the pinning threshold.

(a) Weakly damped swinging of a small specimen may occur when the specimen sits at a position where the mag-

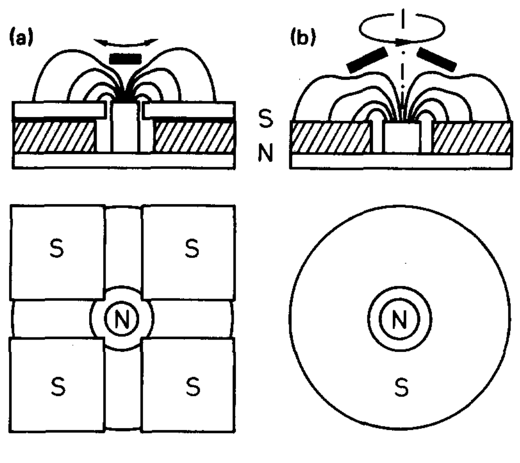

Fig. 3. Various geometries of magnets and types of motion of levitated superconductors (schematic). (a) A superconducting disk oscillating above the center of the magnet shown in Fig. 1. (b) A disk orbiting about the symmetry axis of a magnet with rotational symmetry.

netic field has a minimum or a saddle point. (A magnetic field cannot have isolated maxima in free space.[3]) This is the case, e.g., on the symmetry axis of the magnet as shown in Fig. 3(a). The change of the field is then proportional to the square of the specimen's displacement and thus small-amplitude vibrations are possible as discussed above.

(b) Above magnets with rotational symmetry a superconductor of arbitrary shape can perform circular orbits about the symmetry axis of the magnet without slowing down; see Fig. 3(b). The orbiting specimen will always turn the same face toward the center of the orbit. Its own rotation is thus rigidly coupled to the orbiting motion. In this case the magnetic field at each point on the surface stays constant during the motion, and there is no need for flux to move and thus no energy is dissipated. Even if the superconductor were exactly circular it would not rotate about its own axis independently because the inhomogeneous field on the orbit would brake this rotation. However, torsional oscillations around various axes, and even radial and vertical oscillations, are possible on the orbit provided their amplitudes stay small. Tangential oscillations cannot occur because there is no restoring force along the orbit.

(c) Rotation of a superconductor of any shape about the symmetry axis of the magnet is a special case of (b).

Vertical oscillations are strongly damped since, in order to compensate gravity, the superconductor sinks to a stable position where the magnetic field changes strongly with the height, and any change in height of the superconductor involves flux-line motion. The levitation height z is thus not unique. A type-II superconductor can be pushed down to further stable positions. We will show now that a *continuous range of stable orientations and positions* of levitation and suspension follows from the irreversibility of the magnetization curves.

VII. MAGNETIZATION CURVES

A magnetizable material may be characterized by the magnetization curve $M(H_a)$ of a long cylinder in a longitudinal applied magnetic field $B_a = \mu_0 H_a$. The function $M(H)$ is a material property. The local field is given by $B = \mu_0(H + M)$ where the local magnetization $M = M(H)$ depends on the local field intensity H, which is the sum of H_a and the stray field generated by all magnetized volume elements or, equivalently, by the currents in the sample. In specimens of arbitrary shape, in general, B, H, and M vary spatially even when B_a is constant. However, when the specimen is homogeneous and has the shape of a general ellipsoid, then with B_a also B, H, and M are spatially constant inside the specimen. In particular, when H_a is along one of the principal axes of the ellipsoid, one has

$$H = H_a - NM(H), \tag{1}$$

$$B = B_a + (1 - N)\mu_0 M(H), \tag{2}$$

or

$$\mu_0 M = (B - B_a)/(1 - N). \tag{3}$$

When the material property $M(H)$ is known, H may be eliminated from these implicit equations, and B and $\mu_0 M$ result as functions of B_a.

The constant N in Eq. (1) is called the demagnetization factor (or demagnetizing factor) and $-NM$ is the homogeneous demagnetizing field. The concept of a demagnetization factor N expressed by Eqs. (1)–(3) is useful also for specimens that are only approximately ellipsoidal. For long cylinders in a parallel (perpendicular) field one has $N = 0$ ($N = \frac{1}{2}$), for spheres $N = \frac{1}{3}$, and for thin disks in parallel (perpendicular) field $N = 0$ ($N = 1$). These values follow from symmetry arguments and the fact that the sum of the three N values along the principal axes equals 1.[21] For the general direction of B_a, one may decompose B_a into its Cartesian components along the principal axes and calculate the components of B from Eqs. (1) and (2). In general, this B will not be parallel to B_a but it is still homogeneous. For nonlinear $M(H)$, the equations for the three components of B will be coupled by the condition that M is parallel to H.

For type-I superconductors, Eqs. (1)–(3) are easily solved since $M(H)$ is composed of straight lines: For $\mu_0 H < B_c$ one has $M(H) = -H$, and for $\mu_0 H > B_c$ one has $M(H) = 0$, and thus $B = B_a$. Equations (1)–(3) then yield $\mu_0 M = -B_a/(1 - N)$ for $B_a < (1 - N)B_c$ (Meissner state), $\mu_0 M = -(B_c - B_a)/N$ for $(1 - N)B_c < B_c$ (Landau's intermediate state composed of normal and superconducting lamellas or tubes[4,22]), and $\mu_0 M = 0$ for $B_a > B_c$ (normal conducting state). Note that the intermediate state occurs only for $N > 0$; for $N = 0$, B and M exhibit an abrupt jump at $B_a = B_c$.

Ideal (reversible) magnetization curves for type-II superconductors were calculated from the phenomenological theory of Ginzburg and Landau[23] and from the microscopic theory of Bardeen, Cooper, and Schrieffer (BCS) generalized to inhomogeneous superconducting states by Gor'kov.[24] The theoretical curves $M_{rev}(H)$ nicely agree with the curves measured with nearly pin-free superconductors (dashed curves in Fig. 4) or approximately concluded from the irreversible (hysteretic) magnetization curves of superconductors with weak pinning (solid curves in Fig. 4). For arbitrary N one has $\mu_0 M_{rev} = -B_a/(1 - N)$ for $B_a < (1 - N)B_{c1}$, $\mu_0 M_{rev} = 0$ for $B_a > B_{c2}$, and approximately[16]

$$\mu_0 M_{rev} \approx -C(1 - B_a/B_{c2})B_{c1}, \quad \text{for } B_a > 2B_{c1}, \tag{4}$$

where $C \approx \frac{1}{4}$ to $\frac{1}{2}$. Equation (4) means that for $B_a > 2B_{c1}$ the reversible magnetization of type-II superconductors is

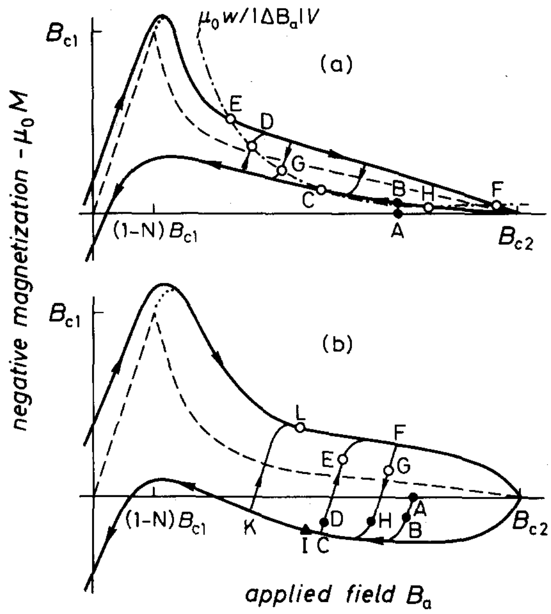

Fig. 4. Magnetization curves of type-II superconductors with (a) weak and (b) strong pinning of flux lines (schematic). The corresponding ideal magnetization curves are shown as dashed lines (pin-free case). The short dotted lines belong to the virgin curves. The big (global) hysteresis loop is symmetric about the origin. Inside this loop an infinite number of small (local) hysteresis loops exists. Levitation is possible where the curve $-\mu_0 M = \mu_0 w/V|dB_a/dz|$ (w = weight and V = volume of the specimen), dash-dotted curve in Fig. 4(a), intersects any of these loops. In Fig. 4(b) the points of levitation (I, E, and G) and suspension (I, D, H, and B) lie on similar curves $-\mu_0 M = \pm \mu_0 w/V|dB_a/dz|$.

typically very small, and the applied field penetrates almost entirely:

$$B_a - B = -(1-N)\mu_0 M_{\rm rev} \ll B_a, \quad \text{for } B_a > 2B_{c1}. \quad (5)$$

Equation (5) means that the Meissner effect in weak-pinning type-II superconductors is typically *very partial*, in particular in high-T_c superconductors, since these exhibit small B_{c1} and large B_{c2}. In addition, the factor $(1-N)$ in Eq. (2) vanishes for flat specimens in a perpendicular field because $N \to 1$ for vanishing thickness; one then has $B = B_a$ for all applied fields. Generally, in the Meissner state the field $B_{\rm eq}$ at the equator of a flat rotational ellipsoid in a perpendicular field is strongly enhanced, $B_{\rm eq} = B_a/(1-N)$. Flux starts to penetrate when $B_{\rm eq}$ reaches B_c in type-I or B_{c1} in type-II superconductors. Thus in a thin disk with $N = 1$ an infinitely small applied field will allow flux to penetrate at the equator.

In superconductors with intermediate [Fig. 4(a)] or strong [Fig. 4(b)] pinning, the Meissner effect, and thus the levitation force, may be considerably larger than in the ideal material. Their magnetization curves exhibit hysteretic behavior; i.e., the magnetization now depends on the "history" whether H_a or the local H was increased or decreased before the present value was reached.

VIII. HYSTERETIC LEVITATION FORCES

From magnetostatics the force **F** on a homogeneously magnetized body with volume V in an inhomogeneous magnetic field $\mathbf{B}_a$ is[21]

$$\mathbf{F} = V(\mathbf{M}\cdot\nabla)\mathbf{B}_a. \quad (6a)$$

Note that $(\mathbf{M}\cdot\nabla)\mathbf{B}_a = \nabla(\mathbf{M}\cdot\mathbf{B}_a)$ since $\nabla\times\mathbf{B}_a = 0$. In particular, when $\mathbf{B}_a$ is along one of the three principal axes of an ellipsoidal specimen located on a symmetry axis of B_a, say, the z axis, then **M** and $\nabla|\mathbf{B}_a|$ are parallel to $\mathbf{B}_a$. Equation (6a) then reduces to

$$F = VM\frac{dB_a}{dz}, \quad (6b)$$

where $\mathbf{F} = \hat{\mathbf{z}}F$, $\mathbf{M} = \hat{\mathbf{z}}M$, and $\mathbf{B}_a = \hat{\mathbf{z}}B_a$ ($\hat{\mathbf{z}}$ = unit vector along z). If, furthermore, the superconductor is small enough that the above assumption of a homogeneous M is sufficiently accurate, then the levitation force on the superconductor can be obtained from the known, shape-dependent magnetization curves $M(B_a)$ [Eqs. (1)–(3)].

The theory of a magnet levitated above a large flat superconductor[25,26] is more difficult since in this case the flux-line density inside the superconductor is not homogeneous. Nevertheless, in both experimental arrangements, pinning leads to hysteretic levitation forces.[27,28] When the hysteresis is sufficiently strong, attractive forces may occur, and free suspension of the superconductor below a magnet becomes possible.[29–31]

In order to visualize how hysteretic levitation and suspension forces come about, we consider a superconducting disk positioned on the symmetry axis above a magnet at height $z \geqslant 0$. For the field at the position of the disk, we assume the model $B_a \sim (z+a)^{-3}$ (the field on the axis of a dipole positioned at $z = -a$, $a > 0$). This gives the field gradient as a function of B_a, $dB_a/dz \sim (z+a)^{-4} \sim B_a^{4/3}$. $F > 0$ means a repulsive force; stable levitation is possible when $F = w$ (w = weight of the sample) and $dF/dz < 0$, corresponding to $dF/dB_a > 0$ since $dB_a/dz < 0$. $F < 0$ means an attractive force; stable suspension is then possible, after turning the arrangement (including the coordinate system) upside down, when $F = -w$ and $dF/dz < 0$ or $dF/dB_a > 0$ hold (the same stability condition as for levitation). From Eq. (6b), with $dB_a/dz < 0$, we see that the disk is repelled if its magnetization is $M < 0$, and attracted if $M > 0$. Moreover, by considering the signs of the two terms $\sim M$ and $\sim dM/dz$ in dF/dz obtained from (6b) one finds that $dM/dB_a < 0$ is a sufficient condition for the stability of levitation, but only a necessary condition for the stability of suspension.

Typical magnetization curves of type-II superconductors are shown in Fig. 4. Note that there is an infinite number of small hysteresis loops connecting the two branches of the large symmetric global hysteresis loop extending from $B_a = -B_{c2}$ to $B_a = B_{c2}$. These small loops are traced out when the direction in which B_a is changing is reversed. Their initial slopes (where they depart from the global loop) are parallel to the low field reversible line, $d\mu_0 M/dB_a = -1/(1-N)$, since the flux is pinned (B = const) for small changes in B_a.

Inside the global hysteresis loop nearly reversible stable levitation or suspension is possible at all positions that satisfy $F = w$ or $F = -w$. These points [such as D and G in Fig. 4(a), E and G or D, H, and B in Fig. 4(b)] lie on the curves $-\mu_0 M = \pm \mu_0 w/V\, dB_a/dz$ for our model field, as indicated explicitly in Fig. 4(a) by the dash-dotted curves. The points where these curves intersect the global hysteresis loop [E, C, H, and F in Fig. 4(a), L and I and two further points near B_{c2} in Fig. 4(b)] are limits of this nearly reversible stability. Inspection of $F(z)$ (Fig. 5) shows that for our model field, points E and C in Fig. 4(a)

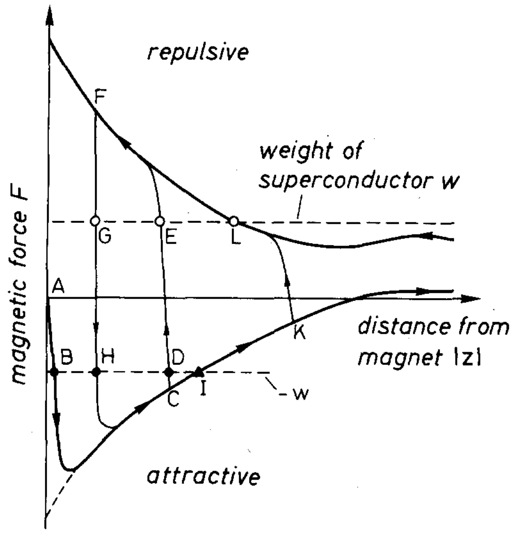

Fig. 5. The force F exerted by a permanent magnet on a small superconductor at distance z (schematic). The curves are calculated from the magnetization of Fig. 4(b) for a model field with $dB_a/dz \sim B_a^{4/3}$. The points A–L in Fig. 4(b) and the present figure correspond to each other. In the upper half of the figure ($F > 0$) z may be interpreted as the levitation height and in the lower half as the suspension depth (after turning the arrangement by 180°). Stable levitation ($F = w$, $dF/dz < 0$) is possible at all heights between the point L and almost $z = 0$ (when pushed onto the magnet and then released, the superconductor lifts a bit). Stable suspension ($F = -w$, $dF/dz < 0$) is possible in the interval between the points B and I.

are also stable with respect to irreversible displacements along the *global* loop, while points F and H in Fig. 4(a) and I in Fig. 5 are not.

The sequence of points in Fig. 4(a) may be interpreted as follows.[28] Point A corresponds (for a certain magnet) to the normal conducting specimen resting on the magnet at $T > T_c$. When cooled below T_c and held fast, the superconductor attains a small negative magnetization since part of its flux is expelled, point B. When released, the superconductor lifts off and reaches a stable levitation height at point C. When lifted and released again, the disk descends a small distance and reaches point D where the levitation is again stable. When depressed and released, the disk arrives at point G. Stable levitation is thus possible between points C and E.

Above a stronger magnet with the same dependence of dB_a/dz on B_a, there exist two continuous ranges of levitation heights for this specimen, between points C and E and between H and F. The levitation heights near B_{c2} in general will not be reached with high-T_c superconductors since B_{c2} may be too large except near T_c where $B_{c2} \sim T_c - T$ is reduced.

A similar interpretation of the sequence of points in the case of stronger pinning [Fig. 4(b) and Fig. 5] may be given. When we cycle the system between points of stable levitation and stable suspension, this means, of course, that in the latter case the attractive magnetic force is such that stable suspension would be achieved if the system were turned by 180°. Points A–L are run through as follows. When the disk is cooled on the magnet it remains there (point A) since its flux density stays constant due to pinning, thus $B = B_a$ and $M = 0$. When it is pulled away from the magnet (i.e., lifted or, in the reversed geometry, released and dropped down), it gains sufficient magnetization by the decreasing B_a and nearly constant B to enable stable suspension at point B. Then the disk is pulled further away to point C and released until it reaches the stable positions D (suspension) or E (levitation). By pressing the disk toward the magnet to point F and releasing it again, one arrives at the stable levitation point G or at the stable suspension point H. By pulling it away again, one passes the unstable suspension point I and reaches K from where the disk may be pushed (or dropped if above the magnet) to the stable levitation point L. The disk thus behaves as if it were bound elastically to a body that slides with dry friction.

IX. CONCLUDING DISCUSSION

In levitated high-temperature superconductors, and generally in all type-II superconductors, the pinning of flux lines by material inhomogeneities leads to conspicuous frictional effects that do not occur in type-I superconductors.

(a) Above magnets with strong transverse gradients the superconductor floats rigidly. Its rotation and its oscillations in all directions are strongly damped.[28]

(b) The position of levitated specimens is not unique. Continuous ranges of positions and orientations exist in which the levitation is stable. The superconductor can be pushed by a finger as if it were embedded in sand. Several specimens may be levitated simultaneously above the same magnet without touching each other (Fig. 1).

(c) If pinning is sufficiently strong, the same superconductor may be levitated above or suspended below a magnet. Suspension originates from the trapping of flux inside the superconductor. In Refs. 29 and 30 suspension was achieved by doping $YBa_2Cu_3O_{6.8}$ with silver oxide in order to enhance pinning, but suspension is also possible with undoped samples[31] and with thallium-based superconductors.[32]

(d) Depending on the temperature-dependent irreversible magnetization curves characterized by $B_{c1}(T)$, $B_{c2}(T)$, and $j_c(B,T)$, different behavior is observed during levitation. For example, one may expect that a superconductor that can be suspended does not lift off when cooled on a magnet (case of Fig. 5). However, when during cooling the pinning strength increases such that the magnetization curve first (near T_c) looks like Fig. 4(a) and then (at 77 K) looks like Fig. 4(b), then a superconductor that can be suspended at 77 K may lift off when cooled on a magnet.

(e) An important effect of flux-line pinning is that it stabilizes the levitation in the *transverse* direction. A permanent magnet floating above another magnet or above a flat type-I superconductor is laterally unstable.[3] It will move sideways or rotate and then be attracted and stick to the magnet. Above a flat type-II superconductor, however, the magnet floats in a stable position since flux pinning removes translational symmetry.[25]

(f) Similarly, the *longitudinal* stability of a suspended type-II superconductor (as the suspension itself) originates from flux pinning. A permanent magnet suspended below another magnet is longitudinally unstable. It appears thus that the suspension of type-II superconductors is the only type of intrinsically stable suspension known at present.

(g) As shown in a recent paper,[33] a ring-shaped magnet, magnetized parallel to its axis, exhibits two points of zero field intensity on its axis, at a distance of 0.4 hole diameters away from its surface. These are the points where the field lines are indifferent whether they should pass through the hole or go around the outer rim to reach the other pole of the magnet. These two free minima of the field can trap any superconductor with a sufficiently strong Meissner effect. Therefore, two superconductors, of type I or type II, may be levitated or suspendend simultaneously to the right and left of a ring magnet, or above or below it. In this case the suspension is caused by the Meissner effect and not by attractive forces.

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