$$\int_0^t \frac{x}{\sqrt{t^2-x^2}} J_0(u'x)dx$$

$$= \int_0^\infty xJ_0(xu')\left[\int_0^\infty \sin(tu)J_0(xu)du\right]dx. \qquad (A12)$$

Interchange the order of integration, and apply Eq. (A1) to obtain

$$\int_0^t \frac{x}{\sqrt{t^2-x^2}} J_0(ux)dx = \frac{1}{u}\sin(tu). \qquad (A13)$$

It then follows that

$$K(t,y,l) = \frac{2}{\pi}\int_0^\infty e^{-lu}\cos(tu)\cos(yu)du, \qquad (A14)$$

from which we obtain Eq. (12). A similar procedure applied to Eq. (3) leads to Eq. (11).

To see that Eq. (4) is satisfied identically by Eq. (7), substitute Eq. (7) into the right-hand side of Eq. (4), interchange the order of integration, and factor out a derivative with respect to t, to obtain the expression

$$\frac{2}{\pi}\int_0^1 f_1(t)\frac{d}{dt}\left[\int_0^\infty \sin(tu)J_0(xu)du\right]dt. \qquad (A15)$$

Since $x>1$, and t is restricted to the interval [0,1], we see that $t<x$. Hence, the expression in brackets is zero, by Eq. (A4). A similar argument, using Eq. (8), applies to Eq. (5).

Finally, we illustrate the charge calculation by finding the charge on the lower disk:

$$Q = 2\pi\int_0^a \epsilon_0\left[\lim_{z\to -0}\left(\frac{\partial\phi}{\partial z}\right) - \lim_{z\to +0}\left(\frac{\partial\phi}{\partial z}\right)\right]\rho\, d\rho. \qquad (A16)$$

Using Eqs. (1) and (7), and the dimensionless variables defined in Eqs. (6):

$$Q = 8\epsilon_0 a\int_0^1\int_0^\infty\int_0^1 xuf_1(t)\cos(ut)J_0(xu)dt\, du\, dx. \qquad (A17)$$

The integral over x gives $J_1(u)$, thus

$$Q = 8\epsilon_0 V_0 a\int_0^1 f(t)\left[\int_0^\infty \cos(tu)J_1(u)du\right]dt. \qquad (A18)$$

The integral in brackets is unity and we obtain the result (14).

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[8]W. R. Smythe, *Static and Dynamic Electricity*, 3rd ed. (McGraw–Hill, New York, 1968), p. 26.

[9]G. Arfken, *Mathematical Methods For Physicists*, 3rd ed. (Academic, New York, 1985), p. 594.

[10]G. N. Watson, *A Treatise on the Theory of Bessel Functions*, 2nd ed. (Cambridge University, Cambridge, 1944), p. 405.

Kamerlingh Onnes and the discovery of superconductivity

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Many papers begin with the statement that Kamerlingh Onnes discovered superconductivity in 1911; one wonders what urged him to do the experiment that led to this discovery. Superconductivity was definitely not foreseen at that moment, and for many years the theory could not even predict it. Hence what drove Kamerlingh Onnes to measure the electrical resistivity at a temperature range where one could not expect to find anything radically new? What were the prevailing theories at that time? This is discussed after a general description of the man, his laboratory, and the state of physics in the preceding years. Conclusion: Although the driving factor in the low temperature resistance work was most likely the need for a reliable and reproducible thermometer, the speculations about what would happen in these regions were based on Kelvin's theory, at least in the initial phase of the work. This theory predicted exactly the opposite of what was found eventually.

In an article in *Physics Today*, Tinkham[1] states that "on countless occasions, I have begun a talk ⋯ by intoning the time-honored sentence 'Superconductivity was discovered in 1911 by Heike Kamerlingh Onnes ⋯' ". This struck a familiar cord with me since almost every term paper on the subject that I had to read started in a similar way. So the question arises, why did Kamerlingh Onnes do this experiment, or, more correctly, why was the experiment undertaken in his laboratory? Kamerlingh Onnes was much too shrewd a man to go for arbitrary extensions of already accomplished results. It seems that at the time the resistance of metals at lower temperatures was fairly well understood, so why go to the extreme? It is clear that superconductivity was not expected, and that, after it was found, it was not understood for a long time. Hence the discovery stands as a lonely tree in the desert, disconnected from the rest of the forest of ideas in physics at that time, or so it seems.

For some time I was at a loss as to where the stimulus for the experiment came from. The papers of Kamerlingh Onnes are very well written and convey the impression that he was a forceful man. In his Address[2] as Rector Magnificus,[3] he very clearly stated what he intended to do (and, amazingly did it all!), but of course there is no reference to superconductivity in this address. There is, however some mention already of the subject of electrical resistance. Kamerlingh Onnes states (Ref. 2, p. 26) that Philipp Lenard concluded from his cathode ray work that the effective center of the atom is concentrated in a tiny fraction of the atomic volume as determined by the kinetic theory of gases. The remaining part Lenard called "dynamids," a term frequently used by Kamerlingh Onnes. In modern terms that would be either the nucleus or the nucleus with the core electrons. Kamerlingh Onnes goes on to reason: "For a moment it has seemed as if at absolute zero the conductivity of metals would increase indefinitely, or at least to an extremely high value. Dewar has found, however, that this is no longer probable." Then he returns to the precipitation idea: "Accordingly, the conductivity, as Kelvin has first expressed it, will at very low temperature reach a maximum and then diminish again till absolute zero is reached, at which point a metal would not conduct at all, any more than glass. This temperature of the maximum of conductivity lies probably sometimes lower than that of liquid hydrogen." This makes it even more interesting to look for the source of the idea behind the experiments on metals at low temperatures.

After Kamerlingh Onnes was appointed in 1882 he stated in his inaugural lecture that physics is capable of improving the well being of the society and proclaimed that this should be accomplished primarily through quantitative measurements. He set out to equip the laboratory on a grand scale; he was an excellent organizer. The city did not yet provide electricity in those days, so he acquired a gas motor and generator and made his own electricity. Pumps and compressors were barely available, so he had them made in the machine shops of the laboratory. There was a need for measuring instruments, so he created an instrument makers' school. There was an even greater demand for glassware (Dewars, McLeod gauges, and connecting tubes, etc.), so he created a glass blowers' school, which became famous in its own right. All currents that had to be measured were sent to a central "measurement room" in which many mirror galvanometers were situated on top of vibration free columns that were separated from the foundations of the building. One should realize that the many announcements in the early literature of the liquefaction of specific gases pertained to not much more than a few drops; Kamerlingh Onnes was planning to make liquid gases by the gallons. A separate hydrogen liquefaction plant was located in a special room with a roof that could be blow off easily. These activities have the flavor of an early Manhattan project. He certainly created one of the first laboratories that was set up in an "industrial" way.

Let us now take a look at the published record. My selection may be of course somewhat biased, since we can no longer pretend to be ignorant of the various consequences of statements made in these papers. In Ref. 4 Kamerlingh Onnes and Clay reported the influence of small admixtures on the resistance in five different metals and gave some general considerations about the matter. This seems to be the last major paper on resistance before the 1911 work. They went down to −258 °C (liquid pentane). In Ref. [5] (February 1911) there is no indication of the upcoming surprise, but there is a change in speculation on the low temperature behavior of the resistance: it is no longer supposed to be merely a minimum, but rather to go to zero. This is justified by the idea that electrons keep moving around, but are less and less hindered by the obstacles, since the vibrators become immovable (i.e., the oscillators freeze, not the electrons). References 6 and 7 contain the discovery of superconductivity. In Ref. 6 (April 1911) the effect is found! Kamerlingh Onnes still makes a linkup with Planck oscillators and states that it "confirmed our forecast" (meaning that the resistance would eventually go to zero). There is no mention of a jump, but in Ref. 7 (May 1911) he curtly admits that the resistance is almost zero below 4.2 K ("sinks below 10^{-4} of the resistance at 0 °C") and that it changes much more quickly than reported before. All explanations are now aborted. In Ref. 8 (December 1911) there is a clear graph of the rapid fall of the resistivity. A very matter of fact tone starts to prevail.

In Ref. 6 there was still some feeble attempt to explain the phenomenon. The reasoning was that according to the Einstein heat capacity theory, the oscillators ceased to oscillate below $kT=h\nu$. Kamerlingh Onnes quoted a spectral line wavelength of mercury in the hope that its value could be linked to the temperature at which superconductivity appeared, but it was soon realized that the jump was extremely sharp and Onnes' attempts to give an explanation ceased for the moment. However there was still one more attempt to understand the new phenomenon, this time by Einstein in 1922.[9] His explanation is somewhat vague and as a consequence I have wondered whether Einstein simply wanted to be nice and help his friends or whether there was some physical basis for his idea. It is certainly not the most quoted work of Einstein. He speculates that the Bohr orbits of the atoms may sometimes overlap and form chains, which comes very close to modern band theory, but retracts this idea in a post-script since it became known meanwhile that the contact between two different superconductive metals was also superconductive.

The decline in resistivity of metals when the temperature was lowered clearly needed further study. This program was both intrinsically interesting as well as relevant to the construction of a good secondary thermometer. It was known that the resistivity was a linear function of the temperature and it became clear that it leveled off at lower temperatures. It was correctly concluded that the height of the plateau depended on the amount of impurities. This was confirmed by a series of experiments with gold, since the amount of admixture in this metal can be easily controlled. Hence it was natural, in order to lower this plateau, to try and purify the metal as much as possible. Since zone melting, in the modern sense, did not exist, it was also natural to use mercury, because this metal could be purified by distillation and the purified liquid was then placed in glass capillaries. When the liquid in these capillaries was frozen it formed a "wire." Moreover, they were aware of the fact that these wires were free of dislocations, to use a modern term, because there is a

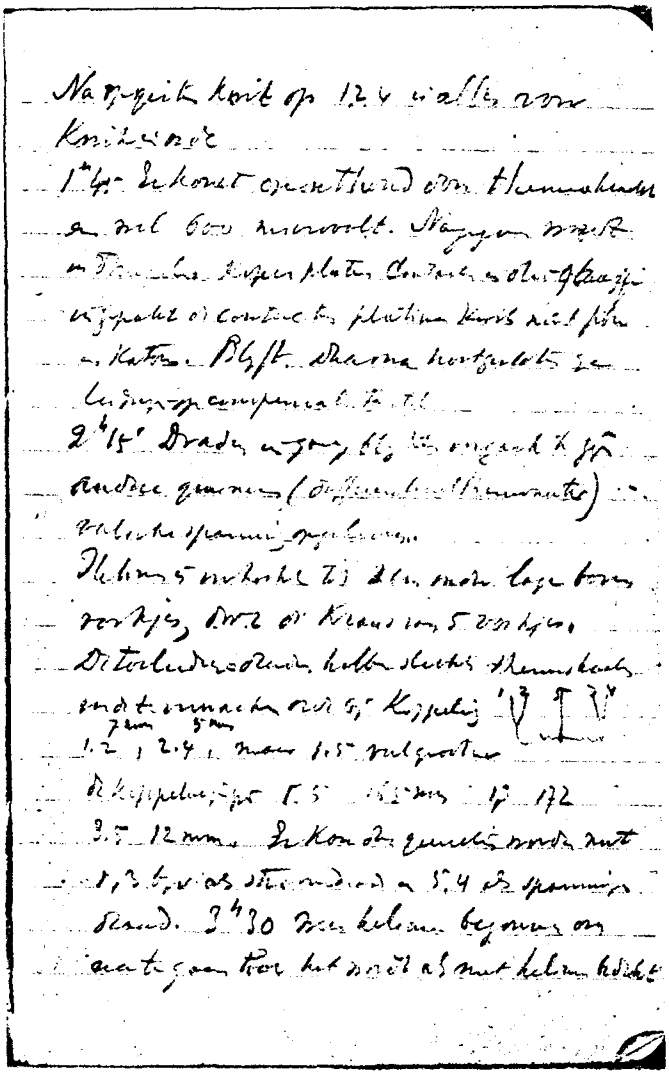

Fig. 1. Reproduction of a page of the notebook used in the laboratory. This is an arbitrary example of the type of information available.

remark in the Proceedings that pulling of wires resulted in increased values of the residual resistance. Hence mercury was the best bet to see how far the linear part of the resistivity curve could be extended to lower temperatures. A linear part at lower temperatures must also have been appealing from the point of view of thermometry.

The people who worked in the physics laboratory had the custom, still existing after the war, to write their results in small notebooks. These notebooks, about 12 by 6 in. with "marmorated" paper covers, were used in every middle class Dutch household to let the baker, grocer, or milkman know how much and what he should drop off at the front door and, hence, these booklets, which now rest in the Boerhaave museum in Leiden, invoke a very homely feeling. The results of the preliminary work with "kwikdraadjes" (small wires of mercury) are reproduced in Fig. 1. However, the booklet in which the famous discovery must have been written down is nowhere to be found in the collection of the museum. As Casimir describes in his memoirs,[10] there are certain aspects surrounding the discovery of superconductivity that are somewhat less glorious,[11] and I wonder whether the loss of the notebook could be the consequence of this.

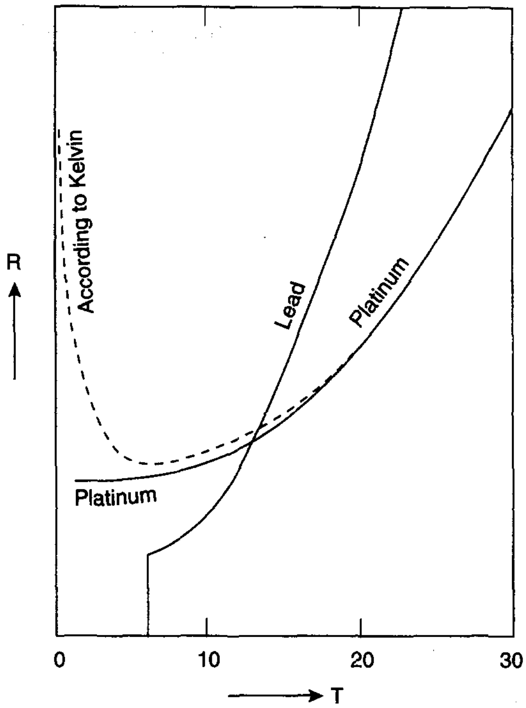

Fig. 2. Three possible curves for the relative electrical resistance R (the ratio between the actual resistance and the resistance at 0 °C) as function of the absolute temperature T. [Taken from Kok (Ref. 13)].

In the biography of Kamerlingh Onnes by van den Handel[12] it is clear that the original expectations must have been quite different, based on a remark by Kelvin, who had theorized that the electrons eventually would condense which would lead to an infinite resistance (see Fig. 2.) Unfortuneately, Kamerlingh Onnes did not often refer to work of other people. In this way he was not alone. Everybody seemed to know anyway what others were doing and had done, thanks to vast personal correspondences and the common practice of multiple publication of the same paper in different languages. It was therefore somewhat of a problem to locate, in the large volume of Kelvin's published work, the remark that the resistivity would become infinite at low temperatures. However, van den Handel[12] refers to the Kamerlingh Onnes' Festschrift,[14] and indeed in the last paper of this publication, written by Crommelin, one finds what obviously had been the prevailing idea before the experiment was undertaken in the Physical Laboratory, as it was called then. The reference I found there is to Kelvin's article: "Aepinus atomized."[15] Moreover since Crommelin's contribution is dedicated to Professor Bosscha,[16] the predecessor of Kamerlingh Onnes, I conclude that Kelvin's paper must have been well known in Leiden at that time.

In this, for modern taste, somewhat curious article Kelvin speculates about the electrical resistivity of metals at low temperatures. He argues as follows. Since it takes very high temperatures to make glasses conductive, a conductivity which he attributes to electrons that are freed from the atoms that make up the substance, and since at room temperature metals contain free electrons, there must be a very low temperature at which the electrons in metals become reattached to the atoms. This is the same reasoning as "corresponding states,"[17] an idea that was frequently and successfully used by Kamerlingh Onnes. Kelvin mentions that this should take place at 1 degree absolute (see Sec. 30 of Ref. 15). Does he use 1 degree absolute simply to indicate what must have been at the time a very low temperature, or was this number based on an estimate? If so, there is no indication in the article of how he came to this value.

Reading Kelvin's article also suggests why Kamerlingh Onnes made the attempt to link the temperature at which the resistance disappeared with the binding energy of electrons in mercury, and ignored the fact that this phenomenon was just the opposite of Kelvin's expectations. There must have been some discussion about this, because at one moment Kamerlingh Onnes writes that the resistance may go either way, to zero or to infinity. It is likely, but cannot be proven, that Kamerlingh Onnes had some doubt about Kelvin's idea to begin with. Possibly he thought that the resistance would go to zero at absolute zero, because in one of the papers he seems to waver between these two extremes.

My claim is that Kamerlingh Onnes, at least at one moment, may have been convinced that the electrons would reattach to the metal ions at low temperatures and that this idea may have been the reason for doing these experiments. However, the experiment could also have been simply guided by the practical wish to be able to extend the range of the resistance thermometer.

There is little doubt that the discovery of superconductivity became the most important discovery of Kamerlingh Onnes' long and extremely fruitful carrier. In the "In Memoriam" Lorentz[18] wrote, he calls it "perhaps the most beautiful pearl of all [of his discoveries]."

I would like to mention that the title of Kelvin's paper[15] gives the misleading impression that he deals with an early Greek, speculating on the atomicity of matter. F.U.T. Aepinus (1742–1802) was a German professor at the Imperial Academy of St. Petersburg,[19] who wrote a book on electricity called "Tentamen," in which he formulated the contemporary state of the theory of electricity. This book remained an important source about this subject until the middle of the 19th century.

I would like to thank Dr. Jaap M. Noothoven van Goor, who provided me with a number of the original articles in the Koninklijke Akademie and other sources. Dr. M. Durieux made me aware of the Einstein paper and told me that in the opinion of Dr. Tuyn the experiment was solely the creation of Kamerlingh Onnes. The Curator of the Rijksmuseum voor de Geschiedenis der Natuurwetenschappen (now Museum Boerhaave) H. Leechburg Auwers, kindly assisted me in searching for the notebook that would have contained the resistance work in 1911. Dr. G.A.C. Veeneman allowed me the publication of Fig. 1. I would also like to thank Professor H. B. G. Casimir for comments on an earlier version of this article. The critique of Dr. Karen Johnson and Dr. N. G. van Kampen was most welcome. Dr. A. N. Gerritsen stressed the importance of the search for reliable resistance thermometers.

[1]M. Tinkham, Phys. Today 22 (March 1986).

[2]Communications from the Physical Laboratory at the University of Leiden. Suppl. 9 No. 85-96 (1904). "The importance of accurate measurements at very low temperatures," address delivered in commemoration of the 329th anniversary of the University of Leiden by the Rector Magnificus: Dr. H. Kamerlingh Onnes. Note: Most papers by Kamerlingh Onnes appeared twice, once in Dutch at the Royal Academy and once in English translation as Communications of the Physical Laboratory. The numbers are the numbers of the Communications, the dates are the dates of the Royal Academy.

[3]The "Rector Magnificus" (Magnificent ruler) is the standard title of the head of a University in the Netherlands, even today. A full professor is elected to fulfill this function for a year. One of his duties is to give an address at the birthday of anniversary of the University: October 3. This is the date of the end of the Spanish siege of the city of Leyden. To commemorate the end of the starvation during this siege, the burgomaster hands out herring and loaves of bread to the population even to day. Prince William of Orange (William the Silent) granted the city the right to found a University out of gratitude that they held out.

[4]Comm. 99c (29 June 1907) Over de verandering van den weerstand der metalen bij zeer lage temperaturen en den invloed, dien kleine bijmengsels hierop hebben. (On the changes in the resistivity in metals at very low temperatures and the influence thereon due to small admixtures) H. Kamerlingh Onnes and J. Clay, Versl. Kon. Akad. Wetenschappen, Amsterdam, Deel XVI.

[5]Comm. 119 (Feb. 1911) Part B. "On the change in the resistance of pure metals at low temperatures etc. III," The resistance of platinum at helium temperatures. H. Kamerlingh Onnes. Same as: Versl. Kon. Akad. Wetenschappen, Amsterdam, Deel XIX, p. 1201.

[6]Comm. 120b (April 1911) "Further experiments with liquid helium. C. On the change of electric resistance of pure metals at very low temperatures etc. IV. The resistance of gold and pure mercury at helium temperatures." Same as: Versl. Kon. Akad. Wetenschappen, Amsterdam, Deel XIX, p. 1479,

[7]Comm. 122b (May 1911) "Further experiments with liquid helium. D. On the change of electric resistance of pure metals at very low temperatures etc. V. The disappearance of the resistance of mercury." Same as: Versl. Kon. Akad. Wetenschappen, Amsterdam, Deel XX, p. 81.

[8]Comm. 124c (Dec. 1911) "Further experiments with liquid helium. G. On the electric resistance of pure metals, etc. VI. On the sudden change in the rate at which the resistance of mercury disappears." Same as: Versl. Kon. Akad. Wetenschappen, Amsterdam, Deel XX, p. 799.

[9]A. Einstein. "Theoretische Bemerkungen zur Supraleitung der Metalle," in Festschrift: Het Natuurkundig Laboratorium der Rijksuniversiteit the Leiden in de Jaren 1904-1922, pp. 429–435; Gedenkboek aangeboden aan H. Kamerlingh Onnes, directeur van het Laboratorium bij gelegenheid van zijn veertigjarig Professoraat op 11 November 1922. Eduard IJdo, Leiden, 1922 (Listed in the L. C. as: Leyden, Rijksuniversiteit K... under QC3.K25.)

[10]H. B. G. Casimir, *Het Toeval van der Werkelijkheid; een halve Eeuw Natuurkunde* (Meulenhoff Informatief, Amsterdam, 1983), pp. 194–95, original American edition: *Haphazard Reality—Half a Century of Science* (Harper and Row, New York, 1983).

[11]The graduate student that ran the experiment in which superconductivity was discovered was Gilles Holst. His request that his name be mentioned in the subsequent publication was ignored. The issue whether Gilles Holst should have been mentioned as codiscover of superconductivity is a question that is described in detail in Ref. 10. However, this is not related to the question I am addressing.

[12]Entry on Kamerlingh Onnes, Heike by J. van den Handel in *Dictionary of Scientific Biography*, edited by C. C. Gillispie (Charles Scribner's Sons, New York, 1973), Vol. VII.

[13]J. A. Kok, *Naar Het Absolute Nulpunt* (Spectrum, Utrecht, 1936).

[14]C. A. Crommelin, "De supra geleidende toestand der metalen," in Het Natuurkundig Laboratorium der Rijksuniversiteit the Leiden in de Jaren 1904-1922, pp. 409–428; Gedenkboek aangeboden aan H. Kamerlingh Onnes, directeur van het Laboratorium bij gelegenheid van zijn veertigjarig Professoraat op 11 November 1922. Eduard IJdo, Leiden, 1922 (Listed in the L. C. as: Leyden, Rijksuniversiteit K... under QC3.K25).

[15]Lord Kelvin, "Aepinus Atomized," Philos. Mag. (Ser. 6) **3**, 257 (1902); from the Jubilee volume presented to Prof. Bosscha in November 1901.

[16]The name Bosscha survives in the annals of the laboratory as the result of the fact that Ehrenfest established a library (by forcing everybody to contribute) in his name: "Leeskamer Bosscha".

[17]The law of corresponding states is the idea of van der Waals that if an equation of state is expressed in reduced variables it should hold for all substances. Reduction means that the temperature, pressure and volume variables are divided by their respective critical values.

[18]H. A. Lorentz, *Collected Papers* (Martinus Nijhoff, the Hague, 1934–1939), Vol. IX.

[19]The reason that this German had a Greek name is that his father translated his surname Hoch, meaning "high", into Greek (*aipeinos* means "lofty"). This was a fashion among intellectuals at that time.