

SéPAG Seminar: Musical Juggling

**From combinatorial modeling
to computer assisted artistic creation**

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Prelude :

What is musical juggling ?

Introduction

Musical juggling consists of playing music by the very act of juggling.



Florent Hivert, Vincent de Lavenère, Nicolas Thiéry
Movie

How I met musical juggling

Year	Study level	Musical Juggling question
2020-2021	M1 MPRI	How to juggle a given melody ?
2022-2023	ARRC Formation	What about jugglers ?
Now	Thesis	The continuation.

Vincent's dream

"Many jugglers playing in a fully-fledged orchestra !"



Training of *Concerto for four jugglers and one piano*,
Scène de recherche, Mai 2023

Vincent's reality

"It takes me xxx hours by hand to find how to juggle xxx, and xxx hours of training to play it proficiently"



Figuring it out...

Goal of the PhD

How to achieve this dream ? What do musical jugglers need ?

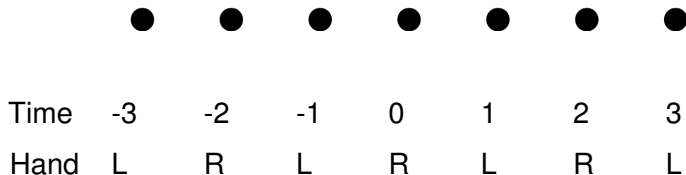
- As creators, to conceive juggling patterns on a music.
- As performers, to train efficiently.

Act I :

**The mathematics
of juggling**

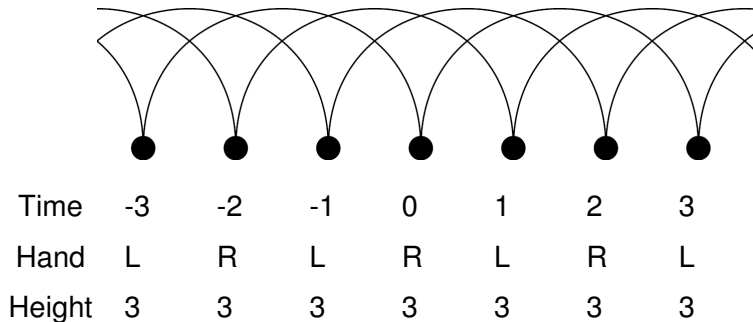
Siteswap notation

B. Polster, *The Mathematics of Juggling*, 2003.

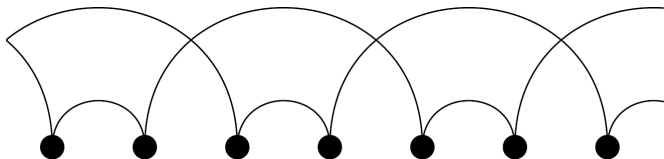


Siteswap notation : 3

B. Polster, *The Mathematics of Juggling*, 2003.



Siteswap notation : 31



Time	-3	-2	-1	0	1	2	3
Hand	L	R	L	R	L	R	L
Height	1	3	1	3	1	3	1

Mathematical definition

Assumptions :

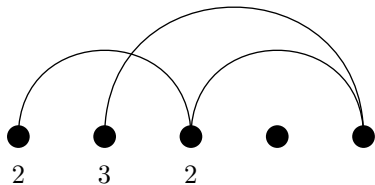
- (Time is discrete).
- (Hands alternate).
- (Juggling patterns are periodic).
- (We juggle forever).
- **Balls are not kept in hand.**
- **At most one ball is caught / thrown at each time tick.**

Definition

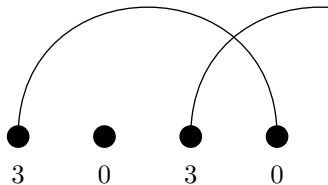
Vanilla siteswap A sequence $(s_i)_{i \in \llbracket 0, p-1 \rrbracket} \in \mathbb{N}^{\llbracket 0, p-1 \rrbracket}$ is jugglable if "one can draw its juggling diagram according to the assumptions above".

Is any sequence jugglable ?

No



Collision !

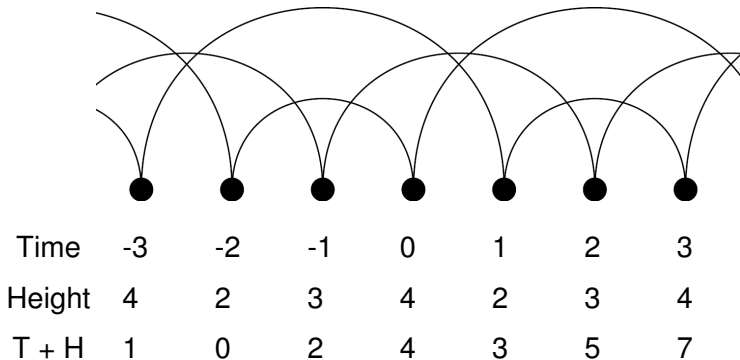


Zero uh-oh !

Jugglability criteria

Theorem (The permutation test)

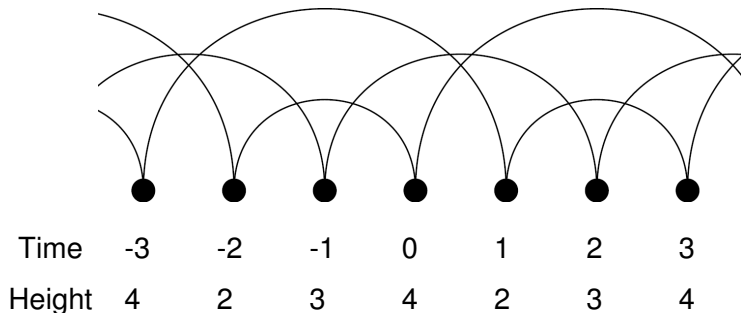
A sequence $(s_i)_{i \in \llbracket 0, p-1 \rrbracket}$ is jugglable iff $(s_i + i \bmod p)_{i \in \llbracket 0, p-1 \rrbracket}$ is a permutation of $\llbracket 0, p-1 \rrbracket$.



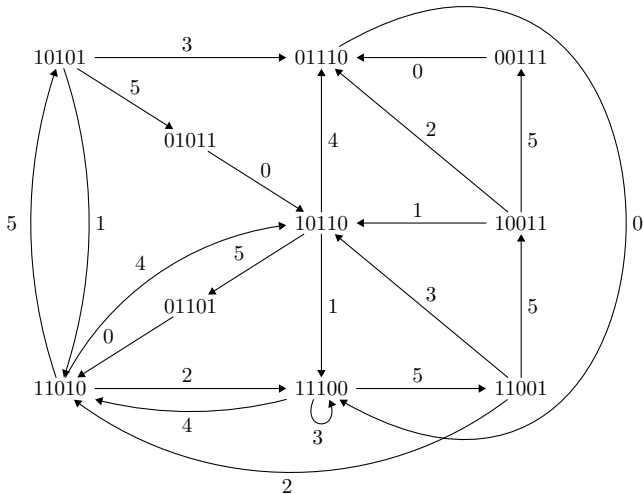
Number of balls

Theorem (The average test)

The number of balls needed to juggle a juggling sequence is its average.

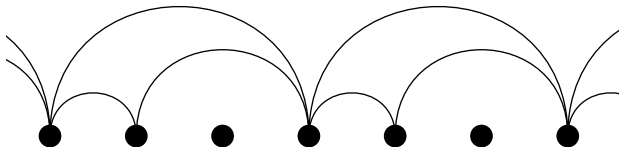


Juggling automata (3 balls, max height 5)



Siteswap extensions : Multiplex juggling

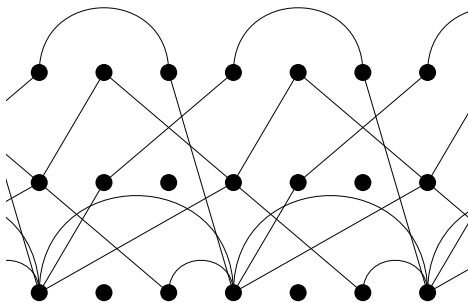
- Multiple throws.
- Multiple catches.



Example : [13]20

Siteswap extensions : Multihand juggling

- Multiple hands.
- Synchronous juggling (hand separation).
- Passing with multiple jugglers.



Example :

$$\begin{pmatrix} 2_0 & 2_1 & 1_2 \\ 1_0 2_2 & 2_0 & 0 \\ [13]_1 3_2 & 0 & 1_2 \end{pmatrix}$$

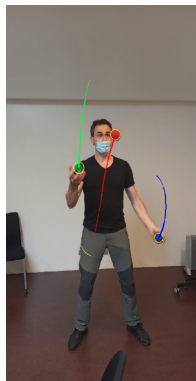
Act II :

Towards musical juggling

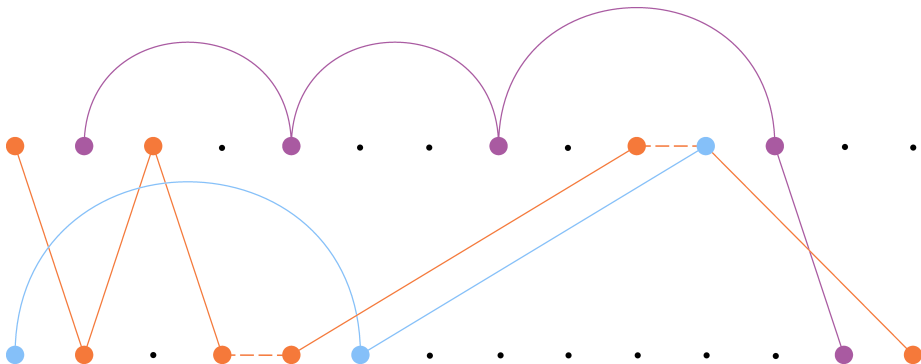
The M1 internship

- Extended siteswap to musical juggling.
- Computed juggling patterns by reducing musical juggling to Exact Cover.
- Visualised juggling patterns with a simulator.
- Experimented with video analysis (tracking).

The image shows a software interface for a juggling simulator. At the top, there is a sequence of numbers: [1 do] [2 do] [3 do] [4 re] [5 m] [7 re] [9 do] [10 m] [11 re] [12 re] [1:]. Below this, there are several control panels. The 'Général' panel includes 'Nombre de mains : 2', 'Nombre maximal de balles dans une main : 3', and 'Hauteur maximale : 5'. The 'Pas' is set to 5, and 'Orientation des mains' is -1, 1. The 'Couleurs' list includes blue, red, green, yellow, purple, cyan, and ma. The 'Prêt' section has two radio buttons: 'Programmation Linéaire (rapide, ne respecte pas les contraintes)' which is selected, and 'Dancing Links (lent, respecte les contraintes sur les mains)'. There are also two checked buttons: 'Résoudre les contraintes' and 'Simuler'. The main area is a 'Simulation' window showing a 2D plot with four colored circles: a green circle at the top, a blue circle on the left, a grey circle at the bottom left, and a red circle on the bottom right. A 'Résultat' tab is visible at the top right of the simulation window. At the bottom of the simulation window, there are playback controls (play, pause, stop, full screen) and a progress bar showing 479.



Musical siteswap ?



Exact Cover

Given :

- $P = \{ \text{🥦}, \text{🍔}, \text{🍟}, \text{🍉} \}$ a set of elements.
- Subsets of P :

$$\begin{aligned} A &= \{ \text{🥦}, \text{🍔} \} \\ B &= \{ \text{🍔}, \text{🍟} \} \\ C &= \{ \text{🥦}, \text{🍟}, \text{🍉} \} \\ D &= \{ \text{🥦}, \text{🍉} \} \\ E &= \{ \text{🍔} \} \end{aligned}$$

$$M = \begin{pmatrix} & \text{🥦} & \text{🍔} & \text{🍟} & \text{🍉} \\ A & 1 & 1 & 0 & 0 \\ B & 0 & 1 & 1 & 0 \\ C & 1 & 0 & 1 & 1 \\ D & 1 & 0 & 0 & 1 \\ E & 0 & 1 & 0 & 0 \end{pmatrix}$$

Question : choose sets such that each element is taken exactly once ?

Question : choose lines of M whose sum is $(1 \ 1 \ 1 \ 1)$?

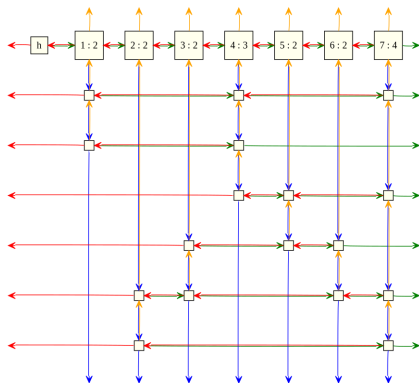
Reduction - General idea

$C(\text{note}, \text{time}, \text{height})$ means "We have to catch *note* at *time*, so we decide to throw it *height* times before."

	Catch do at time 1	Catch ré at time 3
$C(\text{do}, 1, 1)$	1	0
$C(\text{do}, 1, 2)$	1	0
$C(\text{do}, 1, 3)$	1	0
$C(\text{ré}, 3, 1)$	0	1
$C(\text{ré}, 3, 2)$	0	1
$C(\text{ré}, 3, 3)$	0	1

Exact Cover Resolution

- MILP (Mixed Integer Linear Programming)
- Dancing Link (D. E. Knuth, *The Art of Computer Programming, Volume 4B, Combinatorial Algorithms*, 2022).



(Picture credit : [wikipedia](#))

The simulator

[1 do] [2 do] [3 do] [4 re] [5 mi] [7 re] [9 do] [10 mi] [11 re] [12 re] [13 do]

Général Lancers Mains

Nombre de mains :

Nombre maximal de balles dans une main :

Hauteur maximale :

Pas :

Orientation des mains :

Couleurs :

Prêt

Programmation Linéaire (rapide, ne respecte pas les contraintes)

Dancing Links (lent, respecte les contraintes sur les mains)

Résoudre les contraintes Simuler

Simulation Résultat

▶ || ■ ↺

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Act III :

The true goals

Assist the creator

Aim: Create software to compose musical juggling patterns.

Challenge: Handle combinatorial explosion.

Stakes:

- Simple model, complex model.
- Efficient algorithms.
- Interactivity.
- Visualisation.
- Suggest feasible solutions.

Idea: Adapt to each juggler's style and level.



Training



Assist the performer 1

Aim : Help memorise the sequence of throws to make.

Challenge: Cognitive overload.

Context :

- No intuition.
- No "juggling sheet" yet.
- New and narrow field.

Stakes:

- Short-term training and long-term remembering.
- Transmission and notation ?

Ideas:

- Training in VR (slow-mo) ?
- Finding ways to communicate with the juggler (Hearing, touch, ...) ?

Assist the performer 2

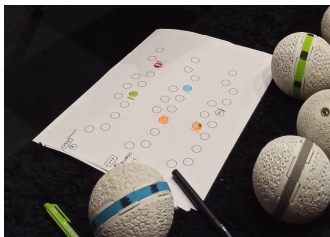
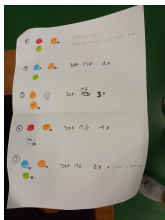
Aim : Help memorise how to take / put back balls from the table.

Stakes :

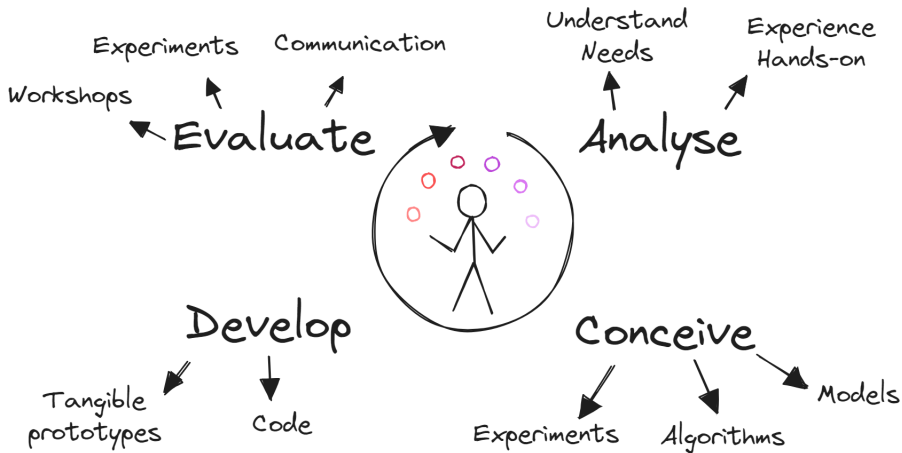
- Retrieval / Deposit method.
- Transmission and notation ?

Ideas:

- Organising balls in order.
- Conceiving notation.



Methodology



Epilogue :

Closing thoughts